Asymmetries and Portfolio Choice

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Abstract

Asset returns are asymmetrically distributed and their correlations change in market up- and downturns. In addition, investor attitudes towards risk indicate an asymmetric treatment of losses versus gains. Motivated by these facts, we examine the portfolio choice of an investor with generalized disappointment aversion preferences who faces returns described by a normal-exponential model. We derive a three-fund separation strategy: The investor allocates wealth to a risk-free asset, a standard mean-variance efficient fund, and an additional fund reflecting return asymmetries. The optimal portfolio is characterized by the investor's endogenous effective risk aversion and implicit asymmetry aversion. We find that disappointment aversion is associated with much larger asymmetry aversion than are standard preferences. Our model explains patterns in popular portfolio advice and provides a reason for shifting from bonds to stocks as the investment horizon increases.

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1 Introduction

Asset returns are asymmetrically distributed and display fatter tails than if they were normally distributed. Correlations between asset returns conditional on downside and upside moves display asymmetric patterns. In particular, correlations between stocks tend to be greater for downside than upside moves, as studied empirically by Ang and Chen (2002) and Hong et al. (2006). In addition, long-term bonds tend to be negatively correlated with stocks conditional on down markets and positively correlated with stocks conditional on up markets (see, e.g., Baele et al., 2010; Campbell et al., 2013; and David and Veronesi, 2013).

Correlated downside movements may undermine diversification benefits precisely when needed. Aversion to these movements can be modeled using utility theories that emphasize investor aversion to downside risk. Such theories include loss aversion, as demonstrated by Kahneman and Tversky (1979) in their prospect theory of choice, rank-dependent expected utility that emerges from the anticipated utility theory of Quiggin (1982), and the disappointment aversion theory of Gul (1991), which has recently been generalized by Routledge and Zin (2010). These preferences are all consistent with the puzzling experimental behavior observed in the Allais (1979) paradox and provide a framework in which investors place different weights on downside losses and upside gains.

We investigate the joint impact of these two types of asymmetries, i.e., in asset returns and in investor attitudes towards risk, on investor portfolio choice. We propose a simple and parsimonious theoretical setup in a static setting, explicitly ruling out any effect that might otherwise arise from purely dynamic channels. It generates portfolio choice behaviors consistent with real-life situations and generates asset demands consistent with popular portfolio recommendations made by financial advisors. Although our setup is static, we consider the effects of different investment horizons.

In modeling asymmetric investor preferences, we focus on the disappointment aversion utility of Gul (1991) and its generalization by Routledge and Zin (2010). This utility specification is axiomatic, normative, and firmly grounded in formal decision theory under uncertainty. Choi et al. (2007) demonstrate that the disappointment aversion utility provides a good interpretation of individual-level data from a series of experiments of subjects facing a portfolio choice problem. Gill and Prowse (2012) provide experimental evidence that people are disappointment averse when they compete. A further advantage of these preferences is that power utility arises as a special case when the degree of disappointment aversion is zero. Our setting is therefore also convenient for studying optimal portfolios when only return asymmetries are present.

To model asymmetries in asset returns we use a normal-exponential model. The model assumes that idiosyncratic security risks follow a multivariate normal distribution, while skewness is generated by a single common factor that follows an exponential distribution, but upon which different securities have different loadings. We find that the proposed model acceptably captures both the skewness of individual asset returns and the coskewness between assets. Moreover, the model can match other key statistical features of the data such as fat tails and asymmetric correlations. A further advantage of using this model is that the normal distribution is a simple special case, when an asset's sensitivity to the common shock is set to zero. Our setting is therefore also convenient for studying optimal portfolios when only preference asymmetry is present.

Our work relates to that of Ang et al. (2005), who consider portfolio choice under disappointment aversion and normally distributed asset returns. We extend their analysis in several directions. First, we consider the generalized disappointment aversion utility in which the reference point distinguishing disappointing from non-disappointing outcomes deviates from the certainty equivalent.[1] Second, we study the effect of asymmetric return distributions supported by data. Third, we derive an analytical solution to the optimal portfolio

[1]In the original version of the utility formulated by Gul (1991), the reference point is the certainty equivalent of the investment. Ang et al. (2005) examine the portfolio choice implications of this original version. Routledge and Zin (2010) introduce a generalized version of the preferences in which the reference point can be set to a given percentage of the certainty equivalent.

problem and our proposed setup easily accommodates multiple risky assets.

Our work also relates to the literature on the effect of skewness on optimal portfolios.[2] Many studies use the third- or fourth-order Taylor's expansion of a differentiable utility function. The asymmetry aversion of the investor is then equivalent to the coefficient of the third-order term in the expansion. This coefficient is usually set to a value implied by some standard utility (as in Jondeau and Rockinger, 2006; Guidolin and Timmermann, 2008; or Martellini and Ziemann, 2010) or chosen ad hoc (as in Harvey et al., 2010). Ghysels et al. (2014) study the effect of conditional quantile-based return asymmetry on the international portfolio choice of investors with power utility. Our approach differs as we study the asymmetry aversion implied by the non-differentiable disappointment aversion utility function, treating the power utility as a special case. Das and Uppal (2004) examine portfolio choice with systemic risks; we consider similar return asymmetries, but treat the investor as having nonstandard preferences and explicitly caring about the downside risk underlying the return asymmetry. Similar to Das and Uppal (2004), we quantify the certainty equivalent cost of ignoring return asymmetry.

We derive an analytical solution to the portfolio choice problem and demonstrate that it leads to a three-fund separation strategy in which the optimal portfolio comprises three funds. The first fund is a risk-free asset while the second is a standard mean-variance efficient fund. The third fund, whose composition is determined by the asymmetry of the risky asset returns, is labeled the "asymmetry-variance" fund. The weight an investor assigns to each

[2] We focus on the portfolio choice implications of skewness; another strand of the literature focuses on its asset pricing implications. For example, Kraus and Litzenberger (1976), Harvey and Siddique (2000), and Dittmar (2002) provide evidence that coskewness is a priced factor in the cross-section of stock returns. The common starting point of these studies is to take the third- or fourth-order Taylor's expansion of the representative investor's utility function and demonstrate that the resulting pricing kernel is a nonlinear function of the return on aggregate wealth. However, these studies do not take a stand on what original utility function is consistent with the estimated price of risk. Dittmar (2002) finds that the pricing kernel implied by the power utility is rejected in the data. Langlois (2013) proposes a setup where the return distribution incorporates both systematic and idiosyncratic asymmetry components and derives the asset pricing implications of the model. He provides evidence that the asymmetry related factors from the model are priced in the returns of various asset classes, but he also does not consider what investor preferences are consistent with the magnitudes of the prices of risk.

fund depends primarily on her preference parameters. Using the analytical solution, we can characterize the effects of asymmetries in returns and preferences.

If there is no asymmetry in returns, they become jointly normal and the asymmetry-variance fund becomes redundant. The investor then chooses to invest in only the risk-free asset and the mean-variance efficient fund. In this case, the effective risk aversion of the investor is enough to describe the optimal portfolio. For a power-utility investor, the effective risk aversion equals the relative risk-aversion coefficient of the utility function. For a disappointment-averse investor, the effective risk aversion depends also on the optimal portfolio, making it endogenous to the optimal choice.[3] We contribute to the literature by deriving the formula for effective risk aversion when the investor has disappointment-aversion preferences. The effective risk aversion provides a convenient way to compare different parameterizations of preferences. We demonstrate that several sets of parameters of the generalized disappointment-aversion preferences (which have two additional parameters compared with the standard power utility) lead to the same effective risk aversion.

If there is asymmetry in the return distribution, the investor allocates some of her wealth to the asymmetry-variance fund. Apart from effective risk aversion, an implicit aversion to asymmetric returns is needed to describe the optimal choice of a given investor. We demonstrate that the asymmetry aversion implied by disappointment-aversion preferences can differ significantly from the values implied by the standard power utility.

Using a calibrated example involving bonds and stocks as risky assets, we illustrate several implications of the model. The asymmetry aversion implied by the power utility is low in magnitude. An investor who is not disappointment averse allocates only a small fraction of her wealth to the asymmetry-variance fund. In other words, standard symmetric preferences imply that return skewness only marginally affects the composition of optimal portfolios. For a disappointment-averse investor, the optimal choice strongly depends on the reference point

[3]The finding that effective risk aversion is endogenous under disappointment aversion is consistent with the discussion presented by Routledge and Zin (2010) and Bonomo et al. (2011).

distinguishing disappointing from non-disappointing outcomes. First, when the reference point equals the certainty equivalent of the investment, a sufficiently disappointment-averse investor does not hold risky securities and instead invests all her wealth in the risk-free asset. This is in line with the findings of Ang et al. (2005). To the contrary, a disappointment-averse investor whose reference point differs from the certainty equivalent always finds it optimal to hold risky assets. Second, when the reference point is lower than the certainty equivalent, the implicit asymmetry aversion is positive and large in magnitude compared with standard preferences. That is, the asymmetry-variance fund becomes important in the optimal portfolio of the disappointment-averse investor, inducing her to shift from the negatively skewed stocks towards the bonds. Increasing disappointment aversion can cause significant shifts in the optimal asset allocation. Third, we examine what happens when the disappointment threshold is higher than the investor's certainty equivalent, a matter the literature has so far largely ignored. A disappointment-averse investor with a high threshold needs a high portfolio return in order not to be disappointed. She prefers stocks over the bonds within her risky portfolio because of their high upside potential. Since stocks are negatively skewed, the implicit asymmetry aversion becomes negative, making it seem that the investor likes negative skewness. However, in this case, the driving force of the choice is not the negative skewness of stocks but their high upside potential.

We also study the effect of investment horizon. Consider a disappointment-averse investor with a disappointment threshold lower than the certainty equivalent. At short investment horizons, the negative skewness deters the investor from holding stocks. However, assuming independent return dynamics, asset returns become more symmetric as the investment horizon increases. Consequently, investors with longer investment horizons hold relatively more stocks than bonds within the risky portfolio. Hence, we provide a reason for shifting from stocks to bonds as the horizon decreases that differs from the reason due to the effective mean reversion in prices (see, e.g., Campbell and Viceira, 2002, 2005). Note also that the data exhibit more persistence in skewness for longer horizons than implied by the independent and identically distributed (IID) assumption. This suggests that return asymmetry might have a larger effect on optimal portfolios for longer investment horizons than in our IID calibration.

Finally, we use the proposed setup to address the portfolio allocation puzzle of Canner et al. (1997). When dividing the portfolio between cash, bonds, and stocks, financial advisors often recommend that conservative investors should allocate more of their risky portfolio (i.e., bonds plus stocks) to bonds, while aggressive investors should allocate more to stocks. Canner et al. (1997) point out that this allocation is inconsistent with the standard portfolio choice theory of Markowitz (1952). The standard model implies that all investors should hold the same composition of risky assets, implying a constant bonds/stocks allocation ratio among different type of investors. Canner et al. (1997) explore various possible explanations for this puzzle and find them all unsatisfactory. We demonstrate that the asymmetry of asset returns coupled with the generalized disappointment aversion of investors can rationalize the portfolio recommendations of these advisors. If conservative investors are more disappointment averse, they should invest a higher fraction of their wealth in the asymmetry-variance fund, and consequently have higher bonds/stocks allocation ratio than aggressive investors.[4]

The rest of the paper is organized as follows. Section 2 introduces the theoretical setup and derives the solution to the portfolio choice problem. Section 3 offers a detailed analysis of the results using a calibrated example. Section 4 concludes the paper.

[4] Other potential explanations for this divergence between theory and practice have been proposed in the literature. Bajeux-Besnainou et al. (2001) explain the puzzle by assuming that the investor's horizon may exceed the maturity of the cash asset. Shalit and Yitzhaki (2003) use conditional stochastic dominance arguments to demonstrate that advisors, acting as agents for numerous clients, recommend portfolios that are not inefficient for all risk-averse investors. Campbell and Viceira (2001) rationalize the popular advice in the context of intertemporal asset allocation models with time-varying expected returns.

2 Theoretical setup

An investor with a generalized disappointment-aversion utility as in Routledge and Zin (2010) can allocate wealth between N risky securities denoted $i = 1, 2, \ldots, N$ and a risk-free asset denoted $i = f$. Similar to Ang and Bekaert (2002), Das and Uppal (2004), Ang et al. (2005), and Guidolin and Timmermann (2008), we consider a finite-horizon setup with utility defined over terminal wealth. Our model for asset returns is set in discrete time.

2.1 Investor attitude towards risk

Generalized disappointment-aversion (GDA) preferences capture the idea that investors care differently about downside losses than about upside gains. The investor's objective is to maximize the utility of the certainty equivalent of the terminal wealth, W_T, at date T. Following Routledge and Zin (2010), the certainty equivalent of the terminal wealth $\mathcal{R}(W_T)$ is implicitly defined by

$$\theta U(\mathcal{R}(W_T)) = E[U(W_T)] - \ell E[(U(\kappa\mathcal{R}(W_T)) - U(W_T)) I(W_T < \kappa\mathcal{R}(W_T))] \ , \tag{1}$$

where $I(\cdot)$ is an indicator function that equals 1 if the condition is met and 0 otherwise, and

$$U(X) = \begin{cases} \frac{X^{1-\gamma}}{1-\gamma} & \text{if } \gamma > 0 \ \text{ and } \gamma \neq 1 \\ \ln X & \text{if } \gamma = 1 \ . \end{cases} \tag{2}$$

The parameter $\gamma > 0$ measures the investor's risk aversion. The parameter $\ell \geq 0$ is the investor's degree of disappointment aversion and $\kappa > 0$ is the percentage of her certainty equivalent below which outcomes are considered disappointing. The parameter θ is defined as $\theta \equiv 1 - \ell(\kappa^{1-\gamma} - 1) I(\kappa > 1)$ and allows one to capture both sides of noncentral disappointment in a single setting. Routledge and Zin (2010) point out that monotonicity imposes the restriction that $\theta > 0$.

If the investor's degree of disappointment aversion is zero ($\ell = 0$), the definition of the certainty equivalent from (1) simplifies to

$$U(\mathcal{R}(W_T)) = E[U(W_T)] \ . \tag{3}$$

In this case, the investor has expected utility (EU) preferences with power utility. When $\ell > 0$, outcomes lower than $\kappa\mathcal{R}(W_T)$ receive an extra weight and lower the investor's certainty equivalent relative to EU. As the objective is to maximize the certainty equivalent, a disappointment-averse investor would like to avoid outcomes below $\kappa\mathcal{R}(W_T)$. The penalty for disappointing outcomes increases with ℓ, so this parameter modulates the importance of disappointment versus satisfaction and can be interpreted as the degree of disappointment aversion. Parameter κ sets the threshold for disappointing outcomes relative to the certainty equivalent. The special case $\kappa = 1$ corresponds to the original disappointment-aversion (DA) preferences of Gul (1991). If $\kappa < 1$, the random future value is considered disappointing if it lies sufficiently below today's certainty equivalent; if $\kappa > 1$, the random future value must be sufficiently far above the certainty equivalent to be considered not disappointing. We demonstrate that different values of κ lead to diverse investor behavior. We refer to an investor for whom $\kappa = 1$ as a DA investor and to an investor for whom $\kappa \neq 1$ as a GDA investor.

Terminal wealth may be written as

$$W_T = W_0 R_{W,T} \ , \tag{4}$$

where $R_{W,T}$ is the gross return on the investor's portfolio over the investment horizon T. Due to the homogeneity of the utility function (2), the following equality holds:

$$\mathcal{R}(W_T) = W_0 \mathcal{R}(R_{W,T}) \ . \tag{5}$$

Ultimately, the investor's objective is simply to maximize the certainty equivalent of the portfolio gross return, $\mathcal{R}(R_{W,T})$, given by

$$\theta U(\mathcal{R}) = E\left[U(R_{W,T})\right] - \ell E\left[(U(\kappa\mathcal{R}) - U(R_{W,T}))\, I(R_{W,T} < \kappa\mathcal{R})\right] \ , \tag{6}$$

where we have used the short-hand notation $\mathcal{R}$ for $\mathcal{R}(R_{W,T})$. Maximizing the certainty equivalent leads to the same solution as does maximizing its logarithm, $\eta \equiv \ln\mathcal{R}$. We demonstrate in Appendix A that the investor's log certainty equivalent is implicitly given by

$$\eta = \begin{cases} \dfrac{1}{1-\gamma}\ln E\left[\exp\left((1-\gamma)\, r_{W,T}\right)\right] & \\ \qquad -\dfrac{1}{1-\gamma}\ln\left(\theta + \ell\kappa^{1-\gamma}\left(1 - E\left[\exp\left((\gamma-1)\, p_{W,T}\right)\right]\right)\right) & \text{if } \gamma > 0 \ \text{ and } \gamma \neq 1 \\ E\left[r_{W,T}\right] - \ell E\left[p_{W,T}\right] + \ell \max\left(\ln\kappa, 0\right) & \text{if } \gamma = 1 \end{cases} , \tag{7}$$

where

$$p_{W,T} \equiv \max\left(\ln\kappa + \eta - r_{W,T}, 0\right) \tag{8}$$

corresponds to the payoff of a European put option on the portfolio's log return $r_{W,T}$, with a strike equal to $\ln\kappa + \eta$, the investor's endogenous threshold of disappointment.

The intuition for (7) is most straightforward when $\gamma = 1$. The investor's log certainty equivalent is a sum of two components. The first component is the log certainty equivalent of the EU investor, while the second component is a downside risk penalty for achieving a portfolio return below the endogenous disappointment threshold. The downside risk is valued as a European put option on the portfolio return with a strike equal to the disappointment threshold. If the portfolio return at the end of the investment period is below the disappointment threshold, the option matures in the money, reducing the utility of the investor. The total cost of downside risk is the expected payoff of this put option, $E\left[p_{W,T}\right]$, times the degree of disappointment aversion, ℓ. Thus, ℓ may be interpretable as the marginal cost of downside risk, as a one-basis-point increase in $E\left[p_{W,T}\right]$ translates into an ℓ-basis-point decrease in the investor's certainty equivalent. When $\gamma \neq 1$, the intuition remains the same. The first component in (7) is the log certainty equivalent of the EU investor. The second part is the downside risk penalty, which is non-positive by definition and a decreasing function of the put option payoff.

2.2 Model of asset returns

We consider a simple extension to the multivariate normal distribution in order to capture the asymmetry in asset returns. Specifically, we assume that log returns on N risky assets are described by the model

$$r_t = \mu - \sigma \circ \delta + (\sigma \circ \delta)\,\varepsilon_{0,t} + \left(\sigma \circ \sqrt{\iota - \delta \circ \delta}\right) \circ \varepsilon_t \ , \tag{9}$$

where μ, σ, and δ are N-dimensional vectors, ι is a vector of ones, and $\circ$ denotes the Schur product (element-wise product) of vectors. The scalar $\varepsilon_{0,t}$ is a common shock across all assets that follows an exponential distribution with a rate parameter equal to one.[5] The N-dimensional vector ε_t represents asset specific shocks and has a multivariate normal distribution, independent of $\varepsilon_{0,t}$, with standard normal marginal densities and correlation matrix Ψ. Parameters μ, σ, Ψ, and δ together describe the return generating model. If $\delta = 0$, then r_t follows a multivariate normal distribution with mean μ, standard deviation vector σ, and correlation matrix Ψ. Hence, our setup conveniently nests the case when asset returns are jointly log normal. In our extended model, N additional parameters in δ are needed compared with the multivariate normal distribution; these additional parameters describe

[5] That is, $\varepsilon_{0,t} \sim \exp(1)$. In a previous version of the paper we considered an alternative model for asset returns, known as the extended skew-normal distribution, where the common shock has a truncated normal distribution. The normal-exponential model in (9) has several advantages. First, the formulas for the return moments are simpler. Second, the extended skew-normal model needs one additional parameter. Third, we are able to derive the exact distribution of multi-period returns for the normal-exponential model, while we have to use approximated distributions if we work with the skew-normal model. Moreover, Adcock and Shutes (2012) show that the normal-exponential model is a certain limiting case of the extended skew-normal distribution, and the two models lead to very similar results in empirical applications.

the asymmetry in returns.

The log return on asset i may be written as

$$r_{i,t} = \mu_i - \sigma_i \delta_i + (\sigma_i \delta_i)\,\varepsilon_{0,t} + \left(\sigma_i \sqrt{1-\delta_i^2}\right) \varepsilon_{i,t} \ . \tag{10}$$

Parameter δ_i, belonging to the interval $(-1, 1)$, determines the sensitivity of the asset return to the exponentially distributed common shock $\varepsilon_{0,t}$. The exponential distribution is suitable for characterizing the occurrence of extreme events, such as large and infrequent losses. For example, the waiting time until the next event in a Poisson-process has an exponential distribution. The Poisson-process is often used to characterize the occurrence of jumps in continuous-time models (see, e.g., Merton, 1976; Bates, 1996; and Broadie et al., 2007). Assets with large negative sensitivities to $\varepsilon_{0,t}$ are subject to large but infrequent negative returns, while assets with large positive sensitivities are subject to large but infrequent positive returns. Model (9) assumes that the occurrence of such extreme movements is simultaneous across assets, so it may be interpretable as a systemic event. In this sense, our discrete-time return dynamics share the properties of the continuous-time dynamics considered by Das and Uppal (2004).

It is straightforward to show that the mean, variance, skewness, and excess kurtosis of $r_{i,t}$ are given by

$$E(r_{i,t}) = \mu_i \ , \qquad Var(r_{i,t}) = \sigma_i^2 \ , \qquad Skew(r_{i,t}) = 2\delta_i^3 \ , \qquad Xkurt(r_{i,t}) = 6\delta_i^4 \ . \tag{11}$$

The correlation and coskewness of the returns of asset i and asset j are

$$\begin{aligned} Corr(r_{i,t}, r_{j,t}) &= \sqrt{1-\delta_i^2}\sqrt{1-\delta_j^2}\psi_{ij} + \delta_i \delta_j \ , \\ Coskew(r_{i,t}, r_{j,t}) &\equiv \frac{E\left[(r_{i,t} - E(r_{i,t}))^2 (r_{j,t} - E(r_{j,t}))\right]}{Var(r_{i,t})\sqrt{Var(r_{j,t})}} = 2\delta_i^2 \delta_j \ . \end{aligned} \tag{12}$$

The formulas in (11) and (12) show how the vector δ characterizes the asymmetry in particular and the non-normality of returns more generally. The parameters of the distribution can be estimated by the generalized method of moments (GMM) using the moments given in (11) and (12). For the main part of the paper, we consider the case when the investment horizon is one period ($T = 1$). That is, asset returns over the investment horizon, r_T, are described by (9). In Section 3.2.3 we also consider the effects of increasing investment horizon.

2.3 The optimal portfolio

The Taylor approximation à la Campbell and Viceira (2002) of the portfolio log return implies that it may be written as

$$r_{W,T} = r_f + w^\top \left(r_T - r_f \iota + \frac{1}{2}\sigma^2 \right) - \frac{1}{2} w^\top \Sigma w \ , \tag{13}$$

where r_f is the risk-free rate, w is the vector of portfolio weights for risky assets, ι is a vector of ones, and σ^2 is the diagonal of the variance-covariance matrix Σ.[6] If individual asset returns are characterized by the return generating model (9), then the portfolio log return is also characterized by (9) with

$$\begin{aligned} \mu_W &= r_f + w^\top \left(\mu - r_f \iota + \frac{1}{2}\sigma^2 \right) - \frac{1}{2} w^\top \Sigma w \ , \\ \sigma_W^2 &= w^\top \Sigma w \ , \\ \delta_W &= \frac{w^\top (\sigma \circ \delta)}{\sigma_W} \ . \end{aligned} \tag{14}$$

Given our setup, the following proposition describes the optimal portfolio.

Proposition 2.1 *The investor's optimal asset allocation may be written as*

$$w = \frac{1}{\tilde{\gamma}} \left(w^{\mathbf{MV}} + \tilde{\chi} w^{\mathbf{AV}} \right) \tag{15}$$

[6] Unreported results indicate that the approximation works well in our calibration exercise.

where

$$w^{\mathbf{MV}} \equiv \Sigma^{-1}\left(\mu - r_f \iota + \frac{1}{2}\sigma^2\right) \quad \text{and} \quad w^{\mathbf{AV}} \equiv \Sigma^{-1}\left(\sigma \circ \delta\right) \ . \tag{16}$$

Analytical expressions for the coefficients $\tilde{\gamma}$ and $\tilde{\chi}$ are given in Appendix B.

Proof. See Appendix B.

Our setup leads to a three-fund separation strategy similar to that of Simaan (1993). The investor allocates her wealth to two risky funds and invests the remainder of her wealth in the risk-free asset. We label the first risky fund, $w^{\mathbf{MV}}$, with "mean variance" because it is the solution to the mean-variance optimal portfolio problem. We label the second risky fund, $w^{\mathbf{AV}}$, with "asymmetry variance" because its composition depends on the asymmetry vector δ and the variance-covariance matrix of the risky asset returns. It is the solution to an asymmetry-variance optimal portfolio problem similar to the mean-variance one.

The weights that the investor assigns to the risky funds are determined by $\tilde{\gamma}$ and $\tilde{\chi}$. These coefficients depend not only on the preference parameters (i.e., γ, ℓ, and κ) but also on the optimal asset allocation, w, itself and the certainty equivalent, η. That is, the coefficients $\tilde{\gamma}$ and $\tilde{\chi}$ and the certainty equivalent η are all endogenous to the model. To solve for these values and for the optimal allocation, w, equations (15) and (7) must be solved simultaneously.

Given the endogenous values of $\tilde{\gamma}$ and $\tilde{\chi}$, the optimal allocation (15) can also be achieved by solving the following mean-variance-asymmetry investment problem:

$$\max_{w} \quad \left(\mu_W - r_f + \frac{1}{2}\sigma_W^2\right) - \frac{\tilde{\gamma}}{2}\sigma_W^2 + \tilde{\chi}\sigma_W\delta_W \ , \tag{17}$$

where μ_W and σ_W^2 are the mean and variance of the portfolio log return given in equation (14), while δ_W describes its asymmetry. Therefore, we can interpret the coefficient $\tilde{\gamma}$ as the *effective risk aversion* and the coefficient $\tilde{\chi}$ as the *implicit asymmetry aversion* of the investor. The finding that effective risk aversion is endogenous under disappointment-aversion preferences is consistent with the discussions presented by Routledge and Zin (2010) and Bonomo et al. (2011) in an intertemporal consumption-based general equilibrium setting. However, unlike these authors, we explicitly derive the formula of effective risk aversion in our partial equilibrium setting. This provides a novel way to quantify the effect of disappointment aversion on the optimal portfolio choice.

The lack of asymmetry in asset returns ($\delta = 0$) implies both $w^{\mathbf{AV}} = 0$ and $\tilde{\chi} = 0$. Hence, the optimal portfolio rule simplifies to

$$w = \frac{1}{\tilde{\gamma}} w^{\mathbf{MV}} \ . \tag{18}$$

When returns are symmetric, investors allocate their wealth between the mean-variance fund and the risk-free asset. Consequently, when observing a particular asset allocation, we cannot determine whether it was chosen by a disappointment-averse or a disappointment-neutral investor. In other words, different combinations of the preference parameter values γ, ℓ, and κ lead to the same $\tilde{\gamma}$. Therefore, the concept of effective risk aversion provides a convenient way to compare the effects of different preferences in the presence of return asymmetries. Comparing the optimal choices of different investors (e.g., EU versus disappointment-averse investors) who have the same effective risk aversion isolates the effect of return asymmetries, as these investors would choose the same portfolios if returns were symmetric.

If the investor has EU preferences, the effective risk aversion is simply the risk aversion for the power utility ($\ell = 0$ implies $\tilde{\gamma} = \gamma$). Disappointment aversion ($\ell > 0$), on the other hand, implies $\tilde{\gamma} > \gamma$; that is, a disappointment-averse investor reduces investment in risky assets, investing a larger fraction of wealth in cash.

3 Empirical application

3.1 Data and parameter estimation

In this section we investigate how investors who differ in their degree of risk aversion and disappointment aversion allocate their wealth among three assets: cash, bonds, and stocks. We estimate return parameters using monthly data for the USA from July 1952 to December 2012, which we obtained from the Center for Research in Security Prices (CRSP). The risk-free rate is the average of the log return on the 30-day Treasury bill from the CRSP Fama Risk-Free Rates file, referred to simply as "cash." The bond return is the return on the 10-year government bond index from the US Treasury and Inflation Series file in CRSP. The stock return is the value-weighted return on the NYSE, NASDAQ, and AMEX. The excess log bond return is the difference between the log return on bonds and the risk-free rate. Similarly, the excess log stock return is the difference between the log return on stocks and the risk-free rate.

Table 1 presents estimation results for the return distribution of the two risky assets as in (9). The parameters are estimated by minimizing the distance between model-implied moments and their sample counterparts, using the generalized method of moments (GMM) with an identity-weighting matrix. The table reports three different GMM estimation results. GMM I is exactly identified and fits the two means, the two volatilities, the correlation, and the two skewness values, where subscript "B" is used for bonds and subscript "S" for stocks. GMM II is overidentified, fitting the two coskewness values in addition to the seven moments considered in GMM I. GMM III fits the same moments as does GMM I but replaces the skewness of the bonds with the coskewness of the stocks relative to the bonds. The top panel of Table 1 presents sample and fitted moments, while the bottom panel presents model parameter estimates.

The sample moments in Table 1 indicate that the monthly excess log bond return has a mean of of 0.13%, a volatility of 2.12%, and a positive skewness of 0.20, while the monthly excess log stock return has a mean of 0.46%, a volatility of 4.26%, and a negative skewness of –0.64. The correlation between the two risky assets is 0.10. All sample moment estimates are significant at conventional significance levels, except for the skewness of the bonds and the coskewness of the bonds relative to the stocks. GMM III is our preferred parameter configuration as it matches the third-order moments of the joint asset distribution that differ significantly from zero. Note that the implied coskewness of the bonds relative to the stocks exactly matches its data counterpart, but that the implied skewness of the bonds is lower than the sample estimate.

The proposed model of asset returns seems to capture all key asset return moments, providing a simple characterization of the return distribution. In contrast, a multivariate normal model of asset returns would assume that the stock skewness and the coskewness of the stocks relative to the bonds equal zero while the data indicate that these higher moments differ significantly from zero. To further illustrate the ability of the model (9) to match key features of asset returns, we use the parameter estimates in Table 1 to compute, via simulations, two additional statistics. The first statistic is the correlation between the bonds and the stocks conditional on the stocks falling below a given quantile of their distribution. The second statistic is the stocks' expected shortfall at a given quantile of the stock distribution. These two statistics are plotted in Figure 1, together with their data counterpart and their analogue computed in the normal distribution. The figure confirms that our return generating model (9) fits these features of the data far better than does the multivariate normal model. Moreover, the fit of the GMM III model is closest to the data, corroborating our choice for the calibration assessment.

3.2 Results

Given the estimated return distribution, the mean-variance fund, $w^{\mathbf{MV}}$, and the asymmetry-variance fund, $w^{\mathbf{AV}}$, can be calculated according to (16). Let us normalize each fund by the absolute value of the sum of its weights. The compositions of the normalized funds are

$$\bar{w}^{\mathbf{MV}} = \frac{w^{\mathbf{MV}}}{|\iota^\top w^{\mathbf{MV}}|} = \begin{pmatrix} 48.43\% \\ 51.57\% \end{pmatrix} \qquad \bar{w}^{\mathbf{AV}} = \frac{w^{\mathbf{AV}}}{|\iota^\top w^{\mathbf{AV}}|} = \begin{pmatrix} 430.53\% \\ -530.53\% \end{pmatrix} . \tag{19}$$

The first entry corresponds to the bonds and the second entry to the stocks. The mean-variance fund assigns a positive weight to both risky assets as they have positive expected excess returns. As the stocks are negatively skewed, the asymmetry-variance fund assigns a negative weight to them, while the bond weight is positive.[7] The optimal portfolio rule (15) can be rewritten as

$$w = \alpha^{\mathbf{MV}} \bar{w}^{\mathbf{MV}} + \alpha^{\mathbf{AV}} \bar{w}^{\mathbf{AV}} \ , \tag{20}$$

where

$$\alpha^{\mathbf{MV}} \equiv \frac{1}{\tilde{\gamma}} \left|\iota^\top w^{\mathbf{MV}}\right| \qquad \text{and} \qquad \alpha^{\mathbf{AV}} \equiv \frac{\tilde{\chi}}{\tilde{\gamma}} \left|\iota^\top w^{\mathbf{AV}}\right| \tag{21}$$

are the weights assigned to the normalized mean-variance and asymmetry-variance funds, respectively. Note that $\alpha^{\mathbf{MV}}$ is a scalar multiple of the investor's effective risk tolerance (i.e., the inverse of effective risk aversion), while $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$ is a scalar multiple of the investor's implicit asymmetry aversion. The optimal investment in cash is $1 - \alpha^{\mathbf{MV}} + \alpha^{\mathbf{AV}}$.

Figure 2 summarizes how investors with different preferences choose their optimal portfolios in our calibrated example. The weight assigned to the mean-variance fund, $\alpha^{\mathbf{MV}}$, is on the horizontal axis and the relative weight of the asymmetry-variance fund, $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$, is on the vertical axis. All curves start at the same point corresponding to an investor for whom $\gamma = 2$ and $\ell = 0$. The dotted line corresponds to the EU investor and shows the effect of

[7] Note that the sum of weights in $\bar{w}^{\mathbf{AV}}$ is –100%. Increasing the weight of $\bar{w}^{\mathbf{AV}}$ in the portfolio corresponds to taking a short position in stocks and a long position in cash and bonds.

increasing γ from 2 to 30. The other curves correspond to disappointment-averse investors with different κ values and show the effect of increasing ℓ from 0 to 3, while keeping γ fixed at 2. Increasing either γ or ℓ leads to higher effective risk aversion, $\tilde{\gamma}$, and consequently to a smaller investment in the mean-variance fund, $\alpha^{\mathbf{MV}}$. Therefore, increasing γ for the EU investor or increasing ℓ for the disappointment-averse investor corresponds to moving to the left along the horizontal axis in Figure 2.

The EU investor increases the relative weight in the asymmetry-variance fund as her risk aversion increases (equivalently, her implicit asymmetry aversion increases). However, the magnitude of the increase is modest: the relative weight of the asymmetry-variance fund in her portfolio is 0.18% at $\gamma = 2$, increasing to only 0.26% at $\gamma = 30$. This emphasizes that EU investors with power utility pay relatively little attention to asymmetries in asset returns in their portfolio choice.

Figure 2 also shows that the DA investor presents a special case. For this investor for whom $\kappa = 1$, $\alpha^{\mathbf{MV}}$ reaches zero at $\ell^* = 0.43$ and remains at zero at all values of $\ell > \ell^*$. That is, a DA investor with high enough disappointment aversion does not hold risky securities at all.[8] However, this result does not hold for disappointment-averse investors for whom $\kappa < 1$. A GDA investor will always find it optimal to hold some risky assets, regardless of the degree of her disappointment aversion. For GDA investors, the relative weight in the asymmetry-variance fund increases considerably as the disappointment aversion ℓ increases; that is, the implicit asymmetry aversion increases with ℓ. The value of $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$ can be as high as 2.6% for GDA investors in Figure 2, which is ten times higher than the value that a highly risk-averse EU investor would choose. Considering the relative proportions of the normalized risky funds in (19), a $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$ value of 2.6% causes a significant shift from stocks to bonds in the optimal portfolio.

While Figure 2 provides an overall picture of the optimal portfolios for investors with

[8] This results holds only in the case of DA investors, for whom the disappointment threshold equals the certainty equivalent of the investment. A disappointment-averse investor whose reference point differs from the certainty equivalent always finds it optimal to hold risky assets.

different preferences, Table 2 presents details of the choices of some investors. The table presents GDA investors for whom $\gamma = 2$, $\ell = 2$, and κ ranges from 0.96 to 1.04. The choice of the DA investor ($\kappa = 1$) is not presented as she chooses not to participate in risky asset markets when $\ell = 2$. In this section, we focus on the cases in which $\kappa \leq 1$, and later discuss cases in which $\kappa > 1$. To highlight the effect of GDA preferences, Panel B of Table 2 describes the choice of the EU investor who has the same effective risk aversion as does the corresponding GDA investor in the same column. Consequently, their investment in the mean-variance fund is exactly the same and the difference between their optimal portfolios comes from the weights they assign to the asymmetry-variance fund.

The GDA investor's threshold for defining disappointing outcomes is determined mainly by κ. The disappointment threshold of GDA investors for whom $\kappa < 1$ is typically negative and decreases by roughly one percentage point when κ decreases by 0.01. For example, the threshold is –1.54% for the investor for whom $\kappa = 0.98$ and –3.51% if $\kappa = 0.96$. While the disappointment threshold of –3.51% might seem low on a monthly basis, note that, given our distributional assumption, the probability of a stock return lower than this threshold is 14.1%, which is not negligible.

The GDA investor has a strong focus on avoiding disappointment. The disappointment probability of the optimal portfolio for the GDA investor for whom $\kappa = 0.98$ is 2.5%, while the probability that the comparable EU investor's portfolio return will fall below the same threshold is 4.3%.[9] In addition, the expected shortfall of the GDA investor's portfolio is –1.98%, while that of the comparable EU investor is –2.15%.[10] To achieve a better disappointment probability and expected shortfall, the GDA investor must reduce her position in assets contributing greatly to the portfolio's left-tail risk. In the $\kappa = 0.98$ example, the

[9]The disappointment probability of the optimal portfolio for the GDA investor is defined as $\pi \equiv Pr\left[r_{W,T} \leq \ln \kappa + \eta\right]$. Note that the comparable EU investor does not become disappointed, but we can calculate the probability that her portfolio return is below the disappointment threshold ($\ln \kappa + \eta$) of the corresponding GDA investor.

[10]The expected shortfall of the portfolio is defined as $es_W \equiv E\left[r_{W,T} \mid r_{W,T} \leq \ln \kappa + \eta\right]$. To calculate a comparable value for the EU investor, we use the disappointment threshold of the corresponding GDA investor.

marginal contribution of the stocks to the portfolio's expected shortfall is –9.24%, almost five times higher than the contribution of the bonds.[11] Therefore, the GDA investor shifts from the stocks towards the bonds in her risky portfolio.

Table 3 shows the resulting optimal portfolios of the same investors presented in Table 2. Staying with the $\kappa = 0.98$ example, both the GDA and the comparable EU investor put 46.9% of their wealth in the mean-variance fund $\bar{w}^{\mathbf{MV}}$, but they assign different weights to the asymmetry-variance fund. The GDA investor assigns 1.19% of her wealth to $\bar{w}^{\mathbf{AV}}$, while the EU investor assigns only one tenth of this amount, 0.11%. As a result, both of them have a similar amount invested in cash, but the GDA investor's bond weight is 4.6 percentage points higher and stock weight is 5.7 percentage points lower than those of the comparable EU investor.

When the disappointment threshold of the GDA investor is lower, she focuses only on avoiding relatively large losses. The difference between the marginal expected shortfalls of the two assets increases as the disappointment threshold declines. Therefore, the shift from stocks to bonds is more pronounced. At $\kappa = 0.96$, the marginal expected shortfalls of stocks and bonds are –9.91% and –1.91%, respectively. The GDA investor's bond weight is 8.5 percentage points higher and stock weight is 10.5 percentage points lower than those of the comparable EU investor.

In general, assets with relatively large marginal expected shortfalls, such as the stocks in our setting, also display relatively large negative skewness and consequently negative weights in the asymmetry-variance fund. For GDA investors for whom $\kappa < 1$, the long position in the asymmetry-variance fund can be interpreted as a hedging demand against downside losses.

3.2.1 Disappointment threshold greater than the certainty equivalent

Routledge and Zin (2010) briefly discuss the possibility of $\kappa > 1$ when introducing GDA preferences. In this case, outcomes must be sufficiently far above the certainty equivalent

[11] The marginal expected shortfall of asset i is $es_i \equiv E\left[r_{i,T} \mid r_{W,T} \leq \ln \kappa + \eta\right]$.

to be non-disappointing. The literature has not yet paid attention to this setting. We demonstrate what $\kappa > 1$ implies for investor behavior. Tables 2 and 3 present the optimal choice even for investors for whom $\kappa > 1$. The most important observation is that the implicit asymmetry aversion is negative, suggesting that a GDA investor with a high disappointment threshold dislikes positive skewness (or, equivalently, likes negative skewness). This result might seem puzzling, but can be well understood.

The disappointment threshold of the investor increases with κ. The GDA investor for whom $\kappa = 1.02$, for example, is satisfied only if the portfolio return is at least 2.51%. To achieve such high returns, she has to rely on assets making a greater contribution to the portfolio's upside potential.[12] The high upside potential ensures that the investor collects a high reward in non-disappointing states. At $\kappa = 1.02$, the marginal contribution of the stocks to the portfolio's upside potential is 7.32%, more than three times higher than the bonds' contribution. Therefore, the GDA investor shifts from bonds to stocks in her risky portfolio by taking a short position in the asymmetry-variance fund. The GDA investor for whom $\kappa = 1.02$ assigns a –1.13% weight to the asymmetry-variance fund, while the comparable EU investor assigns 0.13%. Consequently, the GDA investor has a 6.7-percentage-point higher stock weight and a 5.4-percentage-point-lower bond weight. This leads to a higher probability of no disappointment at the end of the period (i.e., 9.4% for the GDA versus 6.2% for the EU investor) and to higher upside potential for the portfolio (i.e., 3.14% for the GDA versus 3.03% for the EU investor).

The short position in the asymmetry-variance fund can be interpreted as a speculative demand on big upside gains. By preferring the stocks because they have great marginal upside potential, GDA investors for whom $\kappa > 1$ implicitly like negative skewness. This is represented by the negative implicit asymmetry aversion. The contrast between GDA investors for whom $\kappa < 1$ and GDA investors for whom $\kappa > 1$ may be connected to the

[12] The upside potential of the portfolio is the expected return conditional on non-disappointing outcomes: $up_W \equiv E\left[r_{W,T} \mid r_{W,T} > \ln\kappa + \eta\right]$. The marginal upside potential of asset i is $up_i \equiv E\left[r_{i,T} \mid r_{W,T} > \ln\kappa + \eta\right]$.

contrast between hedgers and speculators. Consider, for example, the GDA investors for whom $\kappa = 0.96$ and $\kappa = 1.04$ from Tables 2 and 3. They have very similar effective risk aversions, but their portfolios differ greatly: the investor for whom $\kappa = 0.96$ has a bond/stock allocation ratio of 1.52, while that of the investor for whom $\kappa = 1.04$ is 0.68.

3.2.2 Costs of ignoring skewness

Following Das and Uppal (2004), we quantify the certainty-equivalent cost of ignoring return asymmetries. An investor who ignores asymmetry in the distribution of asset returns and chooses her optimal portfolio as if log asset returns were normally distributed with the same mean and variance-covariance matrix as the true distribution, chooses allocation w'. That suboptimal allocation corresponds to a certainty equivalent, $\mathcal{R}'$, under the true distribution. The cost of ignoring skewness can be measured in absolute terms by

$$\mathcal{R} - \mathcal{R}'$$

or in relative terms by

$$\frac{\mathcal{R}' - R_f}{\mathcal{R} - R_f} \ .$$

The above ratio is the excess certainty equivalent of the suboptimal allocation relative to the excess certainty equivalent of the optimal allocation.

Table 3 shows the absolute and relative costs of ignoring skewness for different investors using the annualized values of the certainty equivalents $\mathcal{R}$ and $\mathcal{R}'$. Note that the absolute measure is multiplied by 1000, so that it indicates the cost for an investor with an initial wealth of \$1000. For EU investors, the certainty-equivalent cost of ignoring skewness is almost negligible, the annualized cost being less than \$0.01 in all the cases. This is in line with the findings of Das and Uppal (2004).[13] The relative measures indicate that EU

[13] Das and Uppal (2004) measure the cost of ignoring systemic risk for EU investors in an international portfolio choice setting. They find similar costs for an investor with a relative risk aversion of 5 and a

investors achieve more than 99.9% of the overall optimal excess certainty equivalent even if they ignore return skewness.

For GDA investors, the cost of ignoring skewness is more substantial. When $\kappa = 0.96$, the cost is \$2.68, considerably higher than for the comparable EU investor. In relative terms, this investor achieves only 89.2% of the optimal excess certainty equivalent if she ignores return skewness. As κ increases, the cost of ignoring return asymmetries declines, but it is still much higher for all GDA investors than for the comparable EU investors.

3.2.3 The effect of time horizon

We next examine the effect of the investment horizon on optimal portfolios assuming that the one-period returns are IID. Let us consider the T-period log return at time t,

$$r_t(T) = \sum_{j=0}^{T-1} r_{t+j} \ . \tag{22}$$

It can be shown that the T-period returns follow[14]

$$r_t(T) = \mu_T - \sqrt{T}(\sigma_T \circ \delta) + (\sigma_T \circ \delta)\varepsilon_{0,T} + \left(\sigma_T \circ \sqrt{\iota - \delta \circ \delta}\right) \circ \varepsilon_T \ , \tag{23}$$

where,

$$\varepsilon_{0,T} \sim \Gamma\left(T, 1/\sqrt{T}\right) \ , \qquad \varepsilon_T \sim N(0, \Psi) \ , \qquad \mu_T = T\mu \ , \qquad \sigma_T = \sqrt{T}\sigma \ . \tag{24}$$

one-year horizon when the portfolio consists of equity indexes of developed countries.

[14] The key result for deriving the return generating model for the T-period returns is that the sum of T IID exponential variables is a random variable that follows a gamma distribution with a shape parameter T. Note also that for $T = 1$, the model in (23) is equivalent to the model in (9), since $\Gamma(1,1) \sim \exp(1)$.

Note that the parameters δ and Ψ do not have a T subscript, since their value is independent of the horizon. The moments of the T-period return of asset i is given by

$$\begin{aligned} E\left(r_{i,t}\left(T\right)\right) &= T\mu_i \ , & Var\left(r_{i,t}\left(T\right)\right) &= T\sigma_i^2 \ , \\ Skew\left(r_{i,t}\left(T\right)\right) &= \frac{2\delta_i^3}{\sqrt{T}} \ , & Xkurt\left(r_{i,t}\left(T\right)\right) &= \frac{6\delta_i^4}{T} \ . \end{aligned} \tag{25}$$

Both the mean and variance of the assets grows by T as the horizon increases. Asset skewness, on the other hand, is scaled by $1/\sqrt{T}$. That is, skewness diminishes as T increases and the distribution of long-horizon returns is closer to normal than the distribution of short-horizon returns. In fact, for large values of T, the distribution of the common shock $\varepsilon_{0,T}$ converges to a normal distribution, hence the asset log returns become jointly normal.

The optimal asset allocation for an investor with horizon T may be written as[15]

$$w_T = \frac{1}{\tilde{\gamma}_T}\left(w^{\mathbf{MV}} + \tilde{\chi}_T w_T^{\mathbf{AV}}\right) \ . \tag{26}$$

Note that $w^{\mathbf{MV}}$ does not have a T subscript. The mean-variance fund has the same composition, irrespective of the investor's horizon. Although the second risky fund, $w_T^{\mathbf{AV}}$, does have a time subscript, it can be shown that $w_T^{\mathbf{AV}} = w^{\mathbf{AV}}/\sqrt{T}$. That is, investors with different horizons will use the same asymmetry-variance fund in their portfolios, only the size of their investment will be different. The parameters describing the risk attitude of the investor, $\tilde{\gamma}_T$ and $\tilde{\chi}_T$, depend on the horizon.

For a given level of effective risk aversion, $\tilde{\gamma}_T$, the only part of the optimal portfolio rule (20) that changes with the investment horizon is $\alpha^{\mathbf{AV}}$, i.e., the weight assigned to $\bar{w}^{\mathbf{AV}}$. To illustrate the effect of horizon in our calibration, for each T we fix $\tilde{\gamma}_T = 5$ (consequently, fix $\alpha^{\mathbf{MV}}$) and calculate the corresponding weight in the asymmetry-variance fund for different investors. For an EU investor, $\tilde{\gamma}_T = 5$ implies $\gamma = 5$. In the case of a GDA investor, a given

[15]Analytical formulas for the horizon-dependent effective risk aversion $\tilde{\gamma}_T$ and implicit asymmetry aversion $\tilde{\chi}_T$ are given in the supplemental appendix.

value of $\tilde{\gamma}_T$ can correspond to different sets of parameter values. We fix $\gamma = 2$ and $\ell = 2$, and choose the value of κ that leads to $\tilde{\gamma}_T = 5$.[16] Note that there are two κ values that lead to $\tilde{\gamma}_T = 5$, one such that $\kappa < 1$ and the other such that $\kappa > 1$. We report results for both cases.

Figure 3A shows how $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$, the relative weight in the normalized asymmetry-variance fund, changes with the horizon. Return asymmetries do not have a large effect on the EU investor's portfolio as her relative weight barely changes with T. For a GDA investor, however, the investment horizon is an important factor determining the optimal portfolio. Over short horizons, GDA investors hold different $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$ ratios than do EU investors due to asymmetries in returns. However, these return asymmetries become less pronounced as the horizon increases. Hence, the $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$ of the GDA investors approaches that of the EU investors. Disappointment aversion (with $\kappa < 1$) together with skewness prompts a shift from bonds to stocks as the investment horizon increases. This is a different mechanism from the often-emphasized effect of mean reversion in prices (see, e.g., Campbell and Viceira, 2002 and 2005).

When returns are IID, the effect of skewness quickly disappears as the horizon increases. However, there is evidence that return skewness does not diminish with the investment horizon as quickly as implied by the IID assumption.[17] To illustrate the effect of persistence in skewness, instead of relying on the IID assumption to calculate T-period returns, we fit our return generating model (9) to returns aggregated over $T = 1, ..., 12$ months. Figure 3B shows the stock's skewness over different horizons. The sample estimates are further from zero than the values implied by the IID assumption. In fact, the skewness of the T-month

[16] We obtain similar results if somewhat different values for ℓ are picked, which suggests that the effects do not depend on one particular set of parameters.

[17] Neuberger (2012) develops an unbiased estimate of the third moment of long-horizon returns from high-frequency returns. He finds that the skewness of US equity index returns does not diminish with the horizon; it actually increases with horizons up to a year and its magnitude is economically important. Ghysels et al. (2014) introduce an asymmetry measure based on conditional quantiles and find that the return asymmetry is more pronounced at the quarterly frequency than at the monthly frequency for the US and many other countries in their sample.

return is greater in magnitude than that of the one-month return for all $T > 1$. Figures 3C and 3D show the optimal stock weight and bond/stock allocation ratio, respectively, for the EU investor and the GDA investor with $\kappa < 1$ in the IID case, and for the same GDA investor with the estimated T-period returns. Since the return asymmetry in the last case does not vanish as the horizon increases, the optimal choice of the GDA investor does not converge to that of the EU investor for holding periods up to one year. This evidence suggests that return asymmetry has a larger effect on optimal portfolios for longer investment horizons than in an IID calibration.

3.2.4 The asset-allocation puzzle

Canner et al. (1997) identified an asset-allocation puzzle. The two-fund separation strategy arising from standard models implies that all investors should hold risky assets in the same proportion, and should change their relative weights in the risky portfolio and in cash only according to their risk appetite. Consequently, all investors should have the same bond/stock ratio in their portfolios. The puzzle, according to Canner et al. (1997), is that financial advisors recommend different ratios for different investors: a high bond/stock ratio for "conservative" investors and a low ratio for "aggressive" investors. Table 4 is adapted from Canner et al. (1997) and presents the recommendations of four financial advisors, together with the assumed asset returns from the original paper.[18] For each advisor, the bond/stock ratio (w_B/w_S) increases as we move from "aggressive" toward "conservative" portfolios.

Given the distributional assumptions in Panel A of Table 4, we can determine the composition of the mean-variance and asymmetry-variance funds using (16). The composition of $\bar{w}^{\mathbf{MV}}$ and $\bar{w}^{\mathbf{MV}}$ is given in the last two columns of Panel A. Using these funds, there is a unique pair of weights, $\alpha^{\mathbf{MV}}$ and $\alpha^{\mathbf{AV}}$, that yields a given recommended portfolio. The last two columns in Panel B show the relative weights in the mean-variance fund ($\alpha^{\mathbf{MV}}$) and

[18]Canner et al. (1997) do not consider asset skewness. We use $s_B = 0.02$ and $s_S = -0.64$ as estimated in our data, but the results are not sensitive to moderate changes in these values.

the asymmetry-variance fund ($\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$) for each portfolio. As we move from "aggressive" towards "conservative" portfolios, the weight in the mean-variance fund decreases, which is consistent with increasing effective risk aversion. At the same time, the relative weight in the asymmetry-variance fund increases, leading to an increase in the bond/stock allocation ratio and consistent with increasing implicit asymmetry aversion. Looking at Figure 2, we see that the $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$ values corresponding to "moderate" and "conservative" portfolios are much higher than those implied by EU preferences, though they are in line with the choice of GDA investors for whom $\kappa < 1$. That is, asset skewness together with increasing disappointment aversion from "aggressive" to "conservative" investors offers an explanation for the asset allocation puzzle of Canner et al. (1997). Finally, note that "aggressive" portfolios are typically associated with negative $\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$ values. As we have discussed earlier, the speculative behavior of GDA investors for whom $\kappa > 1$ might lead to optimal portfolios with this characteristic.

4 Conclusion

We study the joint impact of two types of asymmetries on investor portfolio choice: asymmetries in asset returns and in investor attitudes towards risk. We model asymmetric investor preferences according to generalized disappointment aversion, and asymmetric return distributions using a normal-exponential model.

We find that these two types of asymmetries jointly yield qualitatively different optimal portfolios from those of the standard model in which both these asymmetries are ignored. On the one hand, when asset returns are symmetric, all investors hold the same risky portfolio. Consequently, when observing a particular asset allocation, it is impossible to determine whether it was chosen by a disappointment-averse or a disappointment-neutral investor. On the other hand, standard preferences imply that return asymmetry only marginally affects the composition of optimal portfolios. However, when both asymmetries are taken into account, the composition of the optimal portfolio changes. In our calibrated example, a disappointment-averse investor with a reference point lower than the certainty equivalent of the investment shifts from the negatively skewed stocks towards the bonds in order to avoid the occasional large losses that negatively skew the stock returns. We also demonstrate that the portfolio choice of an investor with longer investment horizon is less affected by skewness.

Apppendix

A The log certainty equivalent

Recall that $r_{W,T} = \ln R_{W,T}$, $\eta = \ln(\mathcal{R})$, and $U(X) = \frac{X^{1-\gamma}}{1-\gamma}$. We can rewrite

$$\begin{aligned} U(\kappa\mathcal{R}) - U(R_{W,T}) &= U(\kappa\mathcal{R})\left(1 - \frac{U(R_{W,T})}{U(\kappa\mathcal{R})}\right) = U(\kappa\mathcal{R})\left(1 - \left(\frac{R_{W,T}}{\kappa\mathcal{R}}\right)^{1-\gamma}\right) \\ &= \kappa^{1-\gamma}U(\mathcal{R})\left(1 - \exp\left((\gamma-1)\left(\ln\kappa + \eta - r_{W,T}\right)\right)\right). \end{aligned} \tag{A.1}$$

Noting that $\forall a, X \in \mathbb{R}$

$$(1 - \exp(aX))\, I(X > 0) = 1 - \exp(aXI(X > 0)) = 1 - \exp(a \max(X, 0))\ , \tag{A.2}$$

equation (A.1) implies

$$E\left[\left(U(\kappa\mathcal{R}) - U(R_{W,T})\right) I(R_{W,T} < \kappa\mathcal{R})\right] = \kappa^{1-\gamma}U(\mathcal{R})\left(1 - E\left[\exp\left((\gamma-1)\, p_{W,T}\right)\right]\right)\ , \tag{A.3}$$

where $p_{W,T} \equiv \max(\ln\kappa + \eta - r_{W,T}, 0)$.

Substituting (A.3) into (6) and solving for $U(\mathcal{R})$, we arrive at

$$\begin{aligned} U(\mathcal{R}) &= \frac{E\left[U(R_{W,T})\right]}{\theta + \ell\kappa^{1-\gamma}\left(1 - E\left[\exp\left((\gamma-1)\, p_{W,T}\right)\right]\right)}. \\ \ln\mathcal{R}^{1-\gamma} &= \ln E\left[R_{W,T}^{1-\gamma}\right] - \ln\left(\theta + \ell\kappa^{1-\gamma}\left(1 - E\left[\exp\left((\gamma-1)\, p_{W,T}\right)\right]\right)\right)\ . \end{aligned} \tag{A.4}$$

This finally leads to the first case in equation (7). The second case in (7) derives directly from the first case by taking the limit and applying l'Hôpital's rule.

B Proof of Proposition 2.1

Equation (7) defines an implicit function

$$G(w,\eta) \equiv -\eta + \frac{1}{1-\gamma}\ln E\left[\exp\left((1-\gamma)\, r_{W,T}\right)\right] - \frac{1}{1-\gamma}\ln\left(\theta + \ell\kappa^{1-\gamma}\left(1 - E\left[\exp\left((\gamma-1)\, p_{W,T}\right)\right]\right)\right) = 0\ . \tag{A.5}$$

Implicit differentiation of (A.5) implies that

$$\frac{\partial\eta}{\partial w} = -\frac{G_1'(w,\eta)}{G_2'(w,\eta)}\ , \tag{A.6}$$

where G_1' is the partial derivative of G with respect to its first argument and G_2' is the partial derivative of G with respect to its second argument. If an optimal allocation policy exists, it satisfies the necessary condition $\frac{\partial\eta}{\partial w} = 0$, implying

$$G_1'(w,\eta) = 0. \tag{A.7}$$

From (A.5),

$$G_1'(w,\eta) = \frac{E\left[\exp\left((1-\gamma)\, r_{W,T}\right)(\partial r_{W,T}/\partial w)\right]}{E\left[\exp\left((1-\gamma)\, r_{W,T}\right)\right]} - \frac{\ell\kappa^{1-\gamma}E\left[\exp\left((\gamma-1)\, p_{W,T}\right)(\partial p_{W,T}/\partial w)\right]}{\theta + \ell\kappa^{1-\gamma}\left(1 - E\left[\exp\left((\gamma-1)\, p_{W,T}\right)\right]\right)}\ . \tag{A.8}$$

Equation (13) implies

$$\frac{\partial r_{W,T}}{\partial w} = \left(r_T - r_f\iota + \frac{1}{2}\sigma^2\right) - \Sigma w \quad \text{and} \quad \frac{\partial p_{W,T}}{\partial w} = -\frac{\partial r_{W,T}}{\partial w} I\left(r_{W,T} < \ln\kappa + \eta\right), \tag{A.9}$$

which substituting into (A.8) yields

$$\begin{aligned} G_1'(w,\eta) = {} & \frac{E\left[\exp\left((1-\gamma)\, r_{W,T}\right) r_T\right]}{E\left[\exp\left((1-\gamma)\, r_{W,T}\right)\right]} + \frac{\nu}{1-\nu}\frac{E\left[\exp\left((1-\gamma)\, r_{W,T}\right) r_T I\left(r_{W,T} < \ln\kappa + \eta\right)\right]}{E\left[\exp\left((1-\gamma)\, r_{W,T}\right) I\left(r_{W,T} < \ln\kappa + \eta\right)\right]} \\ & + \frac{1}{1-\nu}\left(-r_f\iota + \frac{1}{2}\sigma^2 - \Sigma w\right) \end{aligned} \tag{A.10}$$

where

$$\nu \equiv \frac{\ell \kappa^{1-\gamma} \exp\left((\gamma - 1)(\ln \kappa + \eta)\right) E\left[\exp\left((1-\gamma) r_{W,T}\right) I\left(r_{W,T} < \ln \kappa + \eta\right)\right]}{\theta + \ell \kappa^{1-\gamma} E\left[I\left(r_{W,T} < \ln \kappa + \eta\right)\right]} . \tag{A.11}$$

Define

$$M(u, v; T, x) \equiv E\left[\exp\left(u r_{W,T} + v^{\top} r_T\right) I\left(r_{W,T} < x\right)\right] , \tag{A.12}$$

where T denotes the investment horizon. Then, (A.10) can be rewritten as

$$G_1'(w, \eta) = \frac{M_2'(1-\gamma, 0; T, \infty)}{M(1-\gamma, 0; T, \infty)} + \frac{\nu}{1-\nu} \frac{M_2'(1-\gamma, 0; T, \ln \kappa + \eta)}{M(1-\gamma, 0; T, \ln \kappa + \eta)} + \frac{1}{1-\nu}\left(-r_f \iota + \frac{1}{2}\sigma^2 - \Sigma w\right) , \tag{A.13}$$

while the log certainty equivalent and ν can be rewritten as

$$\begin{aligned} \eta = & \frac{1}{1-\gamma} \ln M\left((1-\gamma), 0; T, \infty\right) \\ & - \frac{1}{1-\gamma} \ln\left(\theta + \ell\kappa^{1-\gamma} M(0, 0; T, \ln\kappa + \eta) - \ell\kappa^{1-\gamma} e^{-(1-\gamma)(\ln\kappa+\eta)} M(1-\gamma, 0; T, \ln\kappa + \eta)\right) , \end{aligned} \tag{A.14}$$

and

$$\nu = \frac{\ell\kappa^{1-\gamma} e^{-(1-\gamma)(\ln\kappa+\eta)} M(1-\gamma, 0; T, \ln\kappa + \eta)}{\theta + \ell\kappa^{1-\gamma} M(0, 0; T, \ln\kappa + \eta)} . \tag{A.15}$$

Finding an analytical formula for $M(u, v; T, x)$ and $M_2'(u, v; T, x)$ allows us to calculate all the quantities of interest from (A.13), (A.14), and (A.15). The supplemental appendix contains the derivation of the formulas for these quantities for arbitrary horizon $T \geq 1$. In Proposition 2.1 we are concerned about $T = 1$. Using the results in the supplemental appendix for $T = 1$,

$$M(1-\gamma, 0; 1, \ln\kappa + \eta) = \exp\left((1-\gamma)(\mu_W - \sigma_W \delta_W) + \frac{(1-\gamma)^2 \sigma_W^2 \left(1 - \delta_W^2\right)}{2}\right) \frac{\Phi(c_0) + C}{c_2} \tag{A.16}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution, and

$$C \equiv \begin{cases} \exp\left(\frac{c_2^2+2c_0c_1c_2}{2c_1^2}\right)\Phi\left(-\frac{c_2+c_0c_1}{c_1}\right) & \text{if } c_1 > 0 \\ -\exp\left(\frac{c_2^2+2c_0c_1c_2}{2c_1^2}\right)\Phi\left(\frac{c_2+c_0c_1}{c_1}\right) & \text{if } c_1 < 0 \end{cases}, \tag{A.17}$$

and

$$\begin{aligned} c_0 &\equiv \frac{\ln\kappa + \eta - \mu_W + \sqrt{T}\sigma_W\delta_W - (1-\gamma)\,\sigma_W^2\left(1-\delta_W^2\right)}{\sigma_W\sqrt{1-\delta_W^2}} \\ c_1 &\equiv \frac{-\delta_W}{\sqrt{1-\delta_W^2}} \\ c_2 &\equiv 1-(1-\gamma)\,\sigma_W\delta_W\ . \end{aligned} \tag{A.18}$$

Note also that

$$M(1-\gamma,0;1,\infty) = \exp\left((1-\gamma)(\mu_W - \sigma_W\delta_W) + \frac{(1-\gamma)^2\sigma_W^2\left(1-\delta_W^2\right)}{2}\right)\frac{1}{c_2}\ . \tag{A.19}$$

It is also shown in the supplemental appendix that

$$\frac{M_2'(1-\gamma,0;1,\ln\kappa+\eta)}{M(1-\gamma,0;1,\ln\kappa+\eta)} = \mu + \left(1-\gamma+\frac{\xi_{\Sigma,0}^B}{\Phi(c_0)+C}\right)\Sigma w + \left(\frac{(1-\gamma)^2\sigma_W^2\delta_W^2}{c_2} + \frac{\xi_{a,0}^B}{\Phi(c_0)+C}\right)(\sigma\circ\delta)\ , \tag{A.20}$$

with

$$\begin{aligned} \xi_{a,0}^B &= \exp\left(\frac{c_2^2+2c_0c_1c_2}{2c_1^2}\right)\Phi\left(-\frac{c_2+c_0c_1}{c_1}\right)\left(-c_2-\frac{c_2+c_0c_1}{c_1^2}\right) \\ &\quad + \exp\left(\frac{c_2^2+2c_0c_1c_2}{2c_1^2}\right)\phi\left(-\frac{c_2+c_0c_1}{c_1}\right)\left(\frac{1}{c_1}+c_1\right) - c_1\phi(c_0) \\ \xi_{\Sigma,0}^B &= \exp\left(\frac{c_2^2+2c_0c_1c_2}{2c_1^2}\right)\Phi\left(-\frac{c_2+c_0c_1}{c_1}\right)\frac{c_2}{\sigma_W\delta_W} \\ &\quad + \exp\left(\frac{c_2^2+2c_0c_1c_2}{2c_1^2}\right)\phi\left(-\frac{c_2+c_0c_1}{c_1}\right)\frac{1}{\sigma_W\sqrt{1-\delta_W^2}} - \frac{\phi(c_0)}{\sigma_W\sqrt{1-\delta_W^2}}\ , \end{aligned} \tag{A.21}$$

where $\phi(\cdot)$ is the probability density function of the standard normal distribution. Also,

$$\frac{M_2'(1-\gamma,0;1,\infty)}{M(1-\gamma,0;1,\infty)} = \mu + (1-\gamma)\,\Sigma w + \frac{(1-\gamma)^2\sigma_W^2\delta_W^2}{c_2}(\sigma\circ\delta) \tag{A.22}$$

Substituting (A.20) and (A.22) in (A.13), setting it to zero, and solving for w, we can arrive at the optimal portfolio rule

$$w = \frac{1}{\tilde{\gamma}} \left(\Sigma^{-1} \left(\mu - r_f \iota + \frac{1}{2} \sigma^2 \right) + \tilde{\chi} \Sigma^{-1} \left(\sigma \circ \delta \right) \right) , \tag{A.23}$$

with

$$\begin{aligned} \tilde{\gamma} &= \gamma - \nu \frac{\xi^B_{\Sigma,0}}{\Phi\left(c_0\right) + C} \\ \tilde{\chi} &= \frac{\left(1-\gamma\right)^2 \sigma_W^2 \delta_W^2}{1 - \left(1-\gamma\right) \sigma_W \delta_W} + \nu \frac{\xi^B_{a,0}}{\Phi\left(c_0\right) + C} \end{aligned} , \tag{A.24}$$

which corresponds to (15) in the paper. Note that $\ell = 0$ implies $\nu = 0$.

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Table 1: Parameter estimates

The table presents parameter and moment estimates for the calibration of the model described in (9). The data used for the calibration are monthly log returns on three assets: 30-day Treasury Bills (f), the 10-year government bond index (B), and the value-weighted index of the CRSP stocks (S). The period used is from July 1952 to December 2012. The first two columns present sample moment estimates together with their bootstrapped 95% confidence intervals. The rest of the table shows the results of three different GMM estimations. GMM I is exactly identified and fits the two means (μ), two volatilities (σ), correlation ($corr$), and two skewnesses ($skew$). GMM II is overidentified, fitting the two coskewnesses ($coskew$) in addition to the seven moments considered in GMM I. Finally, GMM III fits the same moments as does GMM I, but the skewness of bonds is replaced with the coskewness of stocks relative to bonds. None of the estimations fits the excess kurtosis values ($xkurt$). The top panel of the table shows sample moments and fitted moments. Values with superscript i are not estimated, but are implied by the fitted distribution. The bottom panel shows additional parameter estimates that are needed to fully describe the model in (9).

	Sample		GMM I		GMM II		GMM III	
	Est	95% c.i.	Est	s.e.	Est	s.e.	Est	s.e.
r_f (%)	0.38							
$\mu_B - r_f$ (%)	0.13	{0.01, 0.26}	0.13	(0.08)	0.13	(0.08)	0.13	(0.08)
$\mu_S - r_f$ (%)	0.46	{0.20, 0.72}	0.46	(0.17)	0.46	(0.17)	0.46	(0.17)
σ_B (%)	2.12	{2.00, 2.24}	2.12	(0.12)	2.12	(0.12)	2.12	(0.12)
σ_S (%)	4.26	{3.99, 4.54}	4.26	(0.22)	4.26	(0.22)	4.26	(0.22)
$corr_{BS}$	0.10	{0.02, 0.18}	0.10	(0.06)	0.10	(0.06)	0.10	(0.06)
$skew_B$	0.20	{-0.06, 0.46}	0.20	(0.23)	0.04	(0.07)	0.02^i	
$skew_S$	-0.64	{-1.06, -0.24}	-0.64	(0.35)	-0.63	(0.35)	-0.64	(0.35)
$coskew_{BS}$	-0.07	{-0.23, 0.10}	-0.30^i		-0.10	(0.04)	-0.07^i	
$coskew_{SB}$	0.21	{0.01, 0.42}	0.44^i		0.26	(0.13)	0.21	(0.14)
$xkurt_B$	1.47	{0.90, 2.02}	0.28^i		0.03^i		0.01^i	
$xkurt_S$	2.40	{0.54, 4.40}	1.33^i		1.28^i		1.33^i	
ψ			0.66	(0.34)	0.41	(0.15)	0.36	(0.17)
δ_B			0.47	(0.18)	0.28	(0.08)	0.22	(0.13)
δ_S			-0.69	(0.13)	-0.68	(0.12)	-0.69	(0.13)

Table 2: Investor attitude towards risk

The table presents detailed information about the optimal portfolio choice of specific investors. For the GDA investors in Panel A, $\gamma = 2$ and $\ell = 2$ are used, while κ varies across columns. The investment horizon is $T = 1$ month. The distribution of asset returns is calibrated using the values reported in panel "GMM III" of Table 1. The log certainty equivalent, expected shortfall, and upside potential are in monthly percentage values. Panel B presents values for a comparable EU investor. The effective risk aversion, $\tilde{\gamma}$, of the investor is exactly the same as that of the GDA investor in the same column, but her implicit asymmetry aversion is the one implied by EU preferences. The π, es_W, and up_W values in Panel B are calculated using the corresponding threshold ($\ln \kappa + \eta$) reported in Panel A.

κ	0.96	0.97	0.98	0.99	1.01	1.02	1.03	1.04
Panel A – GDA investor ($\gamma = 2$ and $\ell = 2$)								
Effective risk aversion, $\tilde{\gamma}$	6.3	8.1	11.9	23.2	20.1	10.5	7.3	5.7
Implicit asymmetry aversion, $\tilde{\chi}$ ($\times 100$)	4.27	4.37	4.46	4.54	-4.27	-3.77	-3.32	-2.92
Log certainty equivalent, η (%)	0.57	0.53	0.48	0.43	0.46	0.53	0.59	0.64
Disappointment threshold, $\ln \kappa + \eta$ (%)	-3.51	-2.51	-1.54	-0.57	1.46	2.51	3.54	4.56
Disappointment probability, π (%)	1.9	2.2	2.5	2.8	89.2	90.6	91.9	92.9
Expected shortfall of portfolio, es_W (%)	-4.36	-3.17	-1.98	-0.80	0.33	0.32	0.33	0.35
Marginal ES of bond, mes_B (%)	–1.91	–1.94	–1.96	–1.98	0.29	0.31	0.33	0.34
Marginal ES of stock, mes_S (%)	–9.91	–9.57	–9.24	–8.93	0.08	0.17	0.25	0.32
Upside potential of portfolio, up_W (%)	0.72	0.65	0.57	0.48	1.80	3.14	4.41	5.63
Marginal UP of bond, mup_B (%)	0.55	0.56	0.56	0.57	2.23	2.36	2.49	2.62
Marginal UP of stock, mup_S (%)	1.05	1.07	1.10	1.12	7.12	7.32	7.50	7.68
Panel B – Comparable EU investor								
Effective risk aversion, $\tilde{\gamma}$	6.3	8.1	11.9	23.2	20.1	10.5	7.3	5.7
Implicit asymmetry aversion, $\tilde{\chi}$ ($\times 10^3$)	0.37	0.40	0.43	0.47	0.46	0.42	0.38	0.35
Disappointment probability, π (%)	3.5	3.9	4.3	4.7	93.1	93.8	94.5	95.1
Expected shortfall of portfolio, es_W (%)	-4.67	-3.41	-2.15	-0.89	0.38	0.39	0.41	0.44
Upside potential of portfolio, up_W (%)	0.86	0.77	0.65	0.53	1.73	3.03	4.28	5.48

Table 3: Optimal portfolios and the cost of ignoring skewness

The table presents optimal portfolio weights and measures for the cost of ignoring return asymmetries. For the GDA investors in Panel A, $\gamma = 2$ and $\ell = 2$ are used, while κ varies across columns. Panel B presents values for a comparable EU investor. The effective risk aversion, $\tilde{\gamma}$, of the investor is exactly the same as that of the GDA investor in the same column, but her implicit asymmetry aversion is that implied by EU preferences. The investment horizon is $T = 1$ month for all investors and the distribution of asset returns is calibrated using the values reported in panel "GMM III" of Table 1. For all investors, the annualized certainty-equivalent cost of ignoring skewness is also presented. The cost in absolute terms is measured as $(\mathcal{R} - \mathcal{R}')$, while in relative terms it is measured as $(\mathcal{R}' - R_f) / (\mathcal{R} - R_f)$, $\mathcal{R}$ being the annualized certainty equivalent of the optimal portfolio. An investor who ignores return asymmetry and chooses her optimal portfolio as if log asset returns were normally distributed with the same mean and variance-covariance matrix as the true distribution, chooses the suboptimal allocation w'. $\mathcal{R}'$ is the annualized certainty equivalent of the suboptimal allocation under the true distribution.

κ	0.96	0.97	0.98	0.99	1.01	1.02	1.03	1.04
Panel A – GDA investor ($\gamma = 2$ and $\ell = 2$)								
Cash weight, w_f (%)	13.2	33.2	54.3	76.5	71.6	45.8	22.3	0.6
Bond weight, w_B (%)	52.4	40.5	27.8	14.3	10.5	20.8	30.8	40.4
Stock weight, w_S (%)	34.4	26.3	17.9	9.1	17.9	33.4	47.0	59.0
MV fund weight, $\alpha^{\mathbf{MV}}$ (%)	89.0	68.5	46.9	24.1	27.7	53.0	76.3	97.8
AV fund weight, $\alpha^{\mathbf{AV}}$ (%)	2.15	1.70	1.19	0.62	-0.67	-1.13	-1.44	-1.62
Certainty-equivalent cost of ignoring skewness								
Relative cost, $\frac{\mathcal{R}' - R_f}{\mathcal{R} - R_f}$ (%)	89.17	89.04	89.01	89.09	96.63	97.19	97.68	98.11
Absolute cost, $\mathcal{R} - \mathcal{R}'$ ($\times 10^3$)	2.677	2.120	1.478	0.765	0.350	0.540	0.619	0.624
Panel B – Comparable EU investor								
Cash weight, w_f (%)	11.2	31.6	53.2	76.0	72.4	47.1	23.9	2.4
Bond weight, w_B (%)	43.9	33.8	23.2	11.9	13.7	26.2	37.7	48.2
Stock weight, w_S (%)	44.9	34.5	23.6	12.1	13.9	26.7	38.5	49.4
MV fund weight, $\alpha^{\mathbf{MV}}$ (%)	89.0	68.5	46.9	24.1	27.7	53.0	76.3	97.8
AV fund weight, $\alpha^{\mathbf{AV}}$ (%)	0.18	0.15	0.11	0.06	0.07	0.13	0.17	0.20
Certainty-equivalent cost of ignoring skewness								
Relative cost, $\frac{\mathcal{R}' - R_f}{\mathcal{R} - R_f}$ (%)	99.96	99.95	99.94	99.93	99.93	99.95	99.96	99.96
Absolute cost, $\mathcal{R} - \mathcal{R}'$ ($\times 10^3$)	0.008	0.007	0.006	0.004	0.004	0.006	0.008	0.008

Table 4: Asset allocations recommended by financial advisors

Panel A of the table presents assumptions regarding the distribution of monthly asset returns based on Table 2 (p. 185) of Canner et al. (1997). Note that Canner et al. (1997) do not report asset skewness, so we use the same values as used in the calibration exercise in the present paper. Panel B of the table presents the recommendations of four financial advisors. The first four columns are taken from Table 1 (p. 183) of Canner et al. (1997). The last two columns present the relative weights in the mean-variance fund $\bar{w}^{\mathbf{MV}}$ ($\alpha^{\mathbf{MV}}$) and the asymmetry-variance fund $\bar{w}^{\mathbf{AV}}$ ($\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$) for each portfolio.

Panel A – Assumed asset returns

	Mean	Std. dev.	Skew	Correlation with		$\bar{w}^{\mathbf{MV}}$	$\bar{w}^{\mathbf{AV}}$
				Bonds	Stocks		
Cash	0.05%						
Bonds	0.18%	2.9%	0.02	1.00	0.23	0.27	22.5
Stocks	0.75%	6.0%	–0.64	0.23	1.00	0.73	–21.5

Panel B – Portfolio recommendations

	Percent of portfolio (%)			$\frac{w_B}{w_S}(\times 100)$	$\alpha^{\mathbf{MV}}(\%)$	$\frac{\alpha^{\mathbf{AV}}}{\alpha^{\mathbf{MV}}}(\times 100)$
	Cash	Bonds	Stocks			
Advisor A						
Conservative	50	30	20	150	49	1.5
Moderate	20	40	40	100	79	1.1
Aggressive	5	30	65	46	95	0.2
Advisor B						
Conservative	20	35	45	78	79	0.8
Moderate	5	40	55	73	94	0.7
Aggressive	5	20	75	27	95	–0.3
Advisor C						
Conservative	50	30	20	150	49	1.5
Moderate	10	40	50	80	89	0.8
Aggressive	0	0	100	0	101	–1.2
Advisor D						
Conservative	20	40	40	100	79	1.1
Moderate	10	30	60	50	90	0.3
Aggressive	0	20	80	25	100	–0.3

Figure 1: Conditional bond–stock correlations and expected stock shortfalls

Figure A plots conditional bond–stock correlations defined as $Corr\left(r_S, r_B | r_S < Q_S\left(q\right)\right)$ if $q \leq 0.5$ and $Corr\left(r_S, r_B | r_S > Q_S\left(q\right)\right)$ if $q > 0.5$, where $Q_S\left(q\right)$ denotes the qth quantile of the stock return distribution. Figure B plots the expected stock return shortfall defined as $E\left[r_S \mid r_S < Q_S\left(q\right)\right]$ and expressed in monthly percentages (%). Both figures display values estimated from the sample (Sample), values simulated from a normal distribution fitted to the data (Normal), and values simulated using model (9) fitted to the data using different GMM estimators (i.e., GMM I, GMM II, and GMM III).

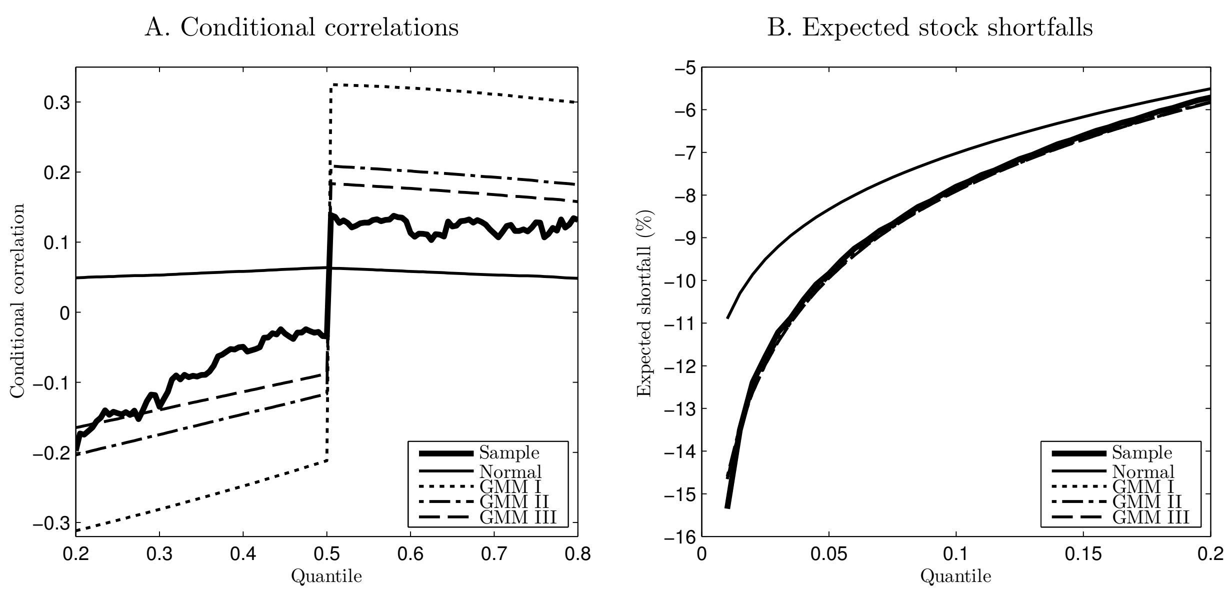

Figure 2: Optimal portfolios for different investors

The figure presents the relative weights in the mean-variance fund ($\alpha^{\mathbf{MV}}$) and the asymmetry-variance fund ($\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$) corresponding to optimal portfolios for investors with different preferences. All curves start at the same point corresponding to the investor for whom $\gamma = 2$ and $\ell = 0$. The line corresponding to the EU investor shows the effect of increasing γ from 2 to 30. Increasing γ leads to higher effective risk aversion and therefore means moving left along the horizontal axis. The other curves correspond to disappointment-averse investors with different κ values (see the legend) and show the effect of increasing ℓ from 0 to 3 while keeping γ fixed at 2. Increasing ℓ leads to higher effective risk aversion and therefore means moving left along the horizontal axis. The investment horizon is $T = 1$ month and the distribution parameters for the asset returns are given in column "GMM III" of Table 1.

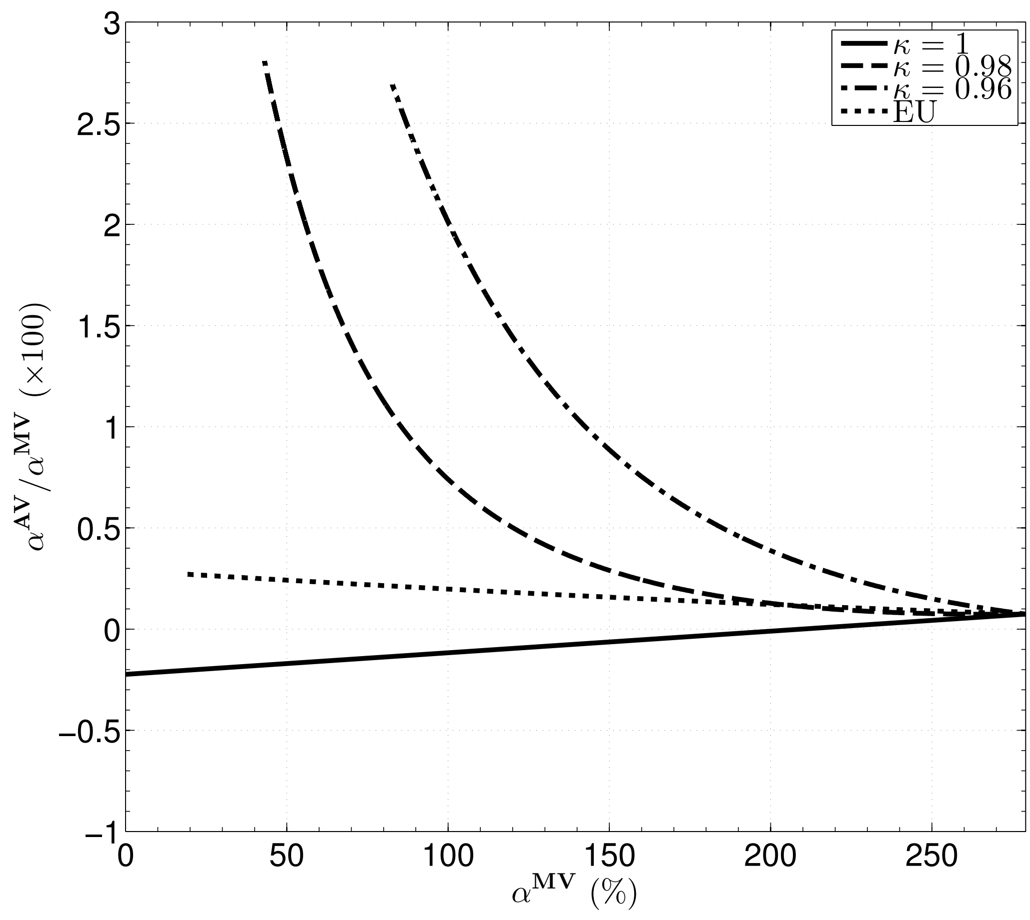

Figure 3: The effect of increasing the investment horizon

Figure A shows how the optimal portfolio changes with the investment horizon if returns are IID. The distribution parameters for the one-period returns are given in column "GMM III" of Table 1, while the T-period parameters are calculated according to (23). The figure shows the relative weight in the asymmetry-variance fund ($\alpha^{\mathbf{AV}}/\alpha^{\mathbf{MV}}$) for the EU ($\ell = 0$) investor and two GDA investors (one for whom $\kappa < 1$ and the other for whom $\kappa > 1$). The preference parameters are chosen so that the effective risk aversion is $\tilde{\gamma} = 5$ for all investors and horizons. Figures B to D compare the IID assumption to the case when the return generating model (9) is fit to returns aggregated over $T = 1, ..., 12$ months. Figure B shows the stock's skewness when returns are aggregated over T months (round markers) and in the IID case (solid line). Figures C and D show the optimal stock weight and bond/stock allocation ratio, respectively, for the EU investor and the GDA investor with $\kappa < 1$ in the IID case, and for the same GDA investor with the estimated T-period returns.