Where's the Kink? Disappointment Events in Consumption Growth and Equilibrium Asset Prices[1]

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Abstract

This paper examines the relation between downside macroeconomic risk and asset prices. Specifically, I test the cross-sectional implications of alternative consumption-based asset pricing models in which investor welfare is defined on deviations from reference points. My results support disappointment aversion preferences in which the reference point is based on the certainty equivalent of consumption growth (Gul (1991), Routledge and Zin (2010)). I find that a single-factor consumption model with disappointment aversion can fit expected returns as accurately as the Fama-French (1993) three-factor model. In contrast, I show that if the reference point is misspecified, downside macroeconomic risk is not priced, and the reference-dependent model cannot fit expected returns.

Keywords: downside macroeconomic risk, consumption-based asset pricing, reference points, generalized disappointment aversion, certainty equivalent, cross-section of expected returns

JEL classification: D51, D91, E21, G12

1. Introduction

A growing literature in economics and finance shows that investors worry about losses more than they enjoy gains, i.e., downside risk matters. In these models with reference-dependent preferences, gains and losses are evaluated relative to a reference point. Since reference points are not directly observed, these models often make assumptions about the location of the reference states. For example, in the loss aversion framework of Kahneman and Tversky (1979), the reference outcome is current wealth (i.e., the status quo), but this choice of reference point is somewhat arbitrary and not fully justified.

The recent literature has employed various reference-dependent models to improve the empirical fit of the traditional asset pricing framework with symmetric preferences.[1] Typically, these models use stock market returns as their main explanatory variable (e.g., downside CAPM)[2] and do not empirically justify their choice of reference points. Moreover, despite the prevalent use of downside CAPMs, reference dependent models that are based on downside consumption risk usually focus on the equity premium and do not provide any results for the cross-section of expected returns.[3] Motivated by these observations, this paper examines the relation between downside consumption risk and the cross-section of expected returns. Specifically, I study reference point formation from an asset pricing perspective, and test whether reference-dependent models with consumption risk alone can explain the cross-section of expected returns.

A key innovation of this paper is that, in pricing the cross-section of expected returns, I consider reference-based models that rely exclusively on consumption risk without resorting to additional behavioral biases (e.g., narrow framing as in Barberis et al. (2001)) or additional explanatory variables (e.g., stock market returns as in Faragó and Tédongap (2014)). Focusing on consumption risk is important for a number of reasons. First, according to the consumption-CAPM literature, consumption growth should be able to explain the cross-section of expected returns (e.g., Lucas (1978),

[1] Epstein and Zin (1990), Benartzi and Thaler (1995), Barberis et al. (2001), Easley and Yang (2012).
[2] Ang et al. (2006), Ostrovnaya et al. (2006), Piccioni (2011), Lettau et al. (2013), Faragó and Tédongap (2014).
[3] Epstein and Zin (1990, 2001), Routledge and Zin (2010), and Bonomo et al. (2011).

Breeden (1979)). Second, and more importantly, despite the large number of results on downside stock market risk, our knowledge about the cross-sectional implications of downside macroeconomic risk is limited.[4] Therefore, by fitting reference-dependent models with consumption risk alone, this paper sheds new light on the relation between the macroeconomy and asset prices.

Another important feature of this paper is that, instead of calibrating a single reference-dependent specification, I estimate a number of models with alternative reference points for gains and losses. This approach allows the data to decide on the significance of the various reference points and identify the one that maximizes the fit of the consumption-based model. By comparing models with alternative reference outcomes, this paper advances our understanding of reference point formation and investor behavior. It also provides answers to the issues raised by Kahneman and Tversky (1991), who state that "A treatment of reference-dependent choice raises two questions: what is the reference state, and how does it affect preferences ?"

The starting point of my theoretical framework is Gul's (1991) model of disappointment aversion. In this model, investor welfare has three characteristics: (i) it is defined based on deviations from reference levels, (ii) it is steeper for losses than for gains (asymmetric utility), and (iii) the reference level is the certainty equivalent of the stochastic payoff. These characteristics imply that disappointment aversion is described by utility functions with kinks (first-order risk aversion).[5]

In this study, I extend Gul's static model to a dynamic setting following the generalized disappointment aversion framework of Routledge and Zin (2010). In the dynamic model, preferences are non-separable across time and the stochastic discount factor is a function of consumption growth and lifetime utility, which is unobservable. Therefore, to estimate the disappointment model, I first obtain an explicit solution for the wealth-consumption ratio in terms of the observable consumption growth. This solution is based on the assumptions that consumption growth is predictable and conditionally homoscedastic. I then use this solution and the Generalized Method of Moments

[4]Epstein and Zin (2001), Campanale, Castro, and Clementi (2010), Routledge and Zin (2010), and Bonomo, Garcia, Meddahi, and Tédongap (2011) focus on the relation between downside consumption risk and the equity premium.

[5]Traditional preference specifications with smooth utility functions (e.g., CRRA or CARA) are usually referred to as second-order risk aversion preferences.

(GMM) to estimate the disappointment aversion discount factor.

The key parameter in the disappointment model is the disappointment aversion coefficient, which measures the asymmetry in investor preferences over gains and losses. To estimate this parameter, I use a GMM system that jointly fits consumption growth moments, as well as Euler equations for the risk-free asset, the stock market, and the six Fama-French portfolios sorted on size and book-to-market. I find that the GMM estimates of the disappointment aversion parameter are positive and statistically significant across all specifications of the disappointment model. For the benchmark specification, in which risk and disappointment aversion coefficients are jointly estimated, the estimate of the disappointment aversion parameter is approximately 7. This result implies that investors penalize losses during disappointment events eight times more than losses during normal times.

An important difference between Gul's (1991) disappointment model and the generalized disappointment aversion framework of Routledge and Zin (2010) is the location of the reference point. In Gul, the reference point is equal to the certainty equivalent, whereas in Routledge and Zin the reference point is a multiple of the certainty equivalent. Based on my empirical results, I cannot reject the hypothesis that the reference point is exactly equal to the certainty equivalent as in Gul's original model. Therefore, my consumption-based asset pricing tests cannot distinguish between Gul's disappointment model and the generalized disappointment framework of Routledge and Zin.

After fitting the disappointment model, I compare its asset pricing performance against alternative consumption-based models (e.g., CRRA and Epstein-Zin). For these tests, I assess the size of the pricing errors and employ several measures of fit such as the cross-sectional R^2, the root mean square error ($RMSE$), and the J-test (Hansen (1982)).

I find that the disappointment model is the only consumption-based model not rejected by the J-test with a cross-sectional R^2 that is nearly 1. In fact, I show that a single-factor consumption-based model with disappointment aversion can explain both the cross-sectional variation ($R^2 = 99\%$) and the level of expected returns ($RMSE = 0.3\%$). Furthermore, the fit of the consumption-based disappointment model is comparable to the fit of the Fama and French (1993) three-factor model ($R^2 = 93\%$). This finding is important because structural models with consumption risk typically perform much worse than return-generated factors.

In addition to comparisons with standard consumption models, I also compare the disappointment framework to models with alternative reference points for gains and losses. First, based on the status quo assumption of Kahneman and Tversky (1979), I consider a specification in which the reference point is equal to zero consumption growth. Second, I assume a model in which the reference point is equal to expected consumption growth as in the expectation framework of Kőszegi and Rabin (2006). Third, I consider a specification in which the reference point is last period's consumption growth. This reference point captures a quickly adapting reference outcome, as implied by the evidence in Arkes et al. (2008).[6] Finally, I propose a specification in which the reference point for consumption growth is a constant parameter estimated by GMM.

My results show that when the reference point is based on either the status quo, the expectation, or the quick adaptation models, then the estimates for the price of downside consumption risk are insignificant. In this case, the reference-dependent model is equivalent to the traditional consumption-based framework with symmetric preferences (no reference points), and cannot explain the cross-section of expected returns. Similar results also hold when the reference point is a constant estimated by GMM, implying that reference points adapt over time.

The explanatory power of consumption-based models with reference-dependent preferences lies in the ability of these models to correctly characterize loss events in consumption growth, i.e., periods during which consumption growth is below the reference point. For the disappointment model, these periods are called disappointment events and happen whenever consumption growth is below its certainty equivalent. During the 1933-2013 period, disappointment events in consumption growth occur with 14% probability and typically happen before or during recessions. Even though disappointment periods do not always overlap recessions, I find that investors are quite sensitive to disappointment events, i.e., periods of much-worse-than-expected consumption growth, and demand high risk premiums for holding assets that underperform during these periods.

[6] Setting the reference point equal to last period's consumption growth can also be motivated by a status quo model in growth rates.

In relation to disappointment events, loss events for the status quo model (zero threshold) occur too rarely (6.2% probability) and ignore a number of loss events in consumption growth that are important for asset prices (e.g., loss events in 1948, 1953, 1956, 1979, 1990, 1999). In contrast, loss events for the expectation and quick-adaptation models, in which the reference points are expected and last period's consumption growth, respectively, happen too often (50% probability). In these models, loss events in consumption growth include periods during which the stock market and the economy is booming. These periods are not related to downside macroeconomic risk. Overall, I find that the alternative reference-based models cannot explain the cross-section of expected returns as accurately as the disappointment model because they do not capture loss events in consumption growth that are important for asset prices.

My empirical findings also contribute to the ongoing discussion in the asset pricing literature regarding the magnitude of the elasticity of intertemporal substitution (EIS). To identify the EIS under the assumption of homoscedastic consumption growth, I include the variance of the risk-free rate in the set of GMM moment conditions. Consistent with the findings of Hall (1988) and Vissing-Jorgensen (2002), the EIS estimates for the consumption-based disappointment model are less than 1 and range between 0.10 and 0.13.

Collectively, my results add to three strands of the asset pricing literature. First, I contribute to the disappointment aversion literature that has considerably grown following the works of Ang, Bekaert, and Liu (2005) and Routledge and Zin (2010).[7] Specifically, Routledge and Zin (2010) and Bonomo et al. (2011) use consumption-based models with disappointment aversion to explain the equity premium but do not provide any results for the cross-section of expected stock returns. In contrast, Ostrovnaya et al. (2006) and Faragó and Tédongap (2014) conduct cross-sectional tests of the disappointment aversion model but use stock market returns as a proxy for returns on aggregate wealth. In this paper, I combine the contributions of Ostrovnaya et al. (2006) and Routledge and Zin (2010) by explicitly solving for the value function in terms of consumption growth, and then

[7]Choi, Fisman, Gale, and Kariv (2007) and Khanapure (2012) examine portfolio choices, and Gill and Prowse (2012) focus on effort provision. Delikouras (2014b) uses disappointment aversion to explain the credit spread puzzle. Dolmas (2014) combines disappointment aversion with rare disasters, while Schreindorfer (2014) uses disappointment aversion to price put options.

applying generalized disappointment aversion to the cross-section of expected returns.

An explicit solution for the value function in terms of consumption growth is significant for a number of reasons. First, characterizing the pricing kernel in terms of aggregate consumption forces the disappointment model to confront asset pricing moments using macroeconomic data alone. Second, I find that, at the annual frequency, the consumption-based disappointment model fits expected returns better than the disappointment model in which stock market returns are used as a proxy for aggregate wealth returns. This is consistent with the evidence in Lustig et al. (2013) who show that stock market wealth has different properties from aggregate wealth. In addition to improving the fit of the disappointment model, the consumption-based solutions facilitate the joint identification of the risk aversion coefficient, the disappointment aversion parameter, and the EIS. Finally, unlike previous results that rely on calibrations, by estimating the disappointment model, I can identify actual disappointment events in consumption growth and examine the relation between these events and macroeconomic conditions (e.g., recessions).

This paper also adds to the empirical asset-pricing literature. First, in terms of estimation, my empirical framework jointly fits expected returns and consumption growth moments via a system of GMM moment conditions. By jointly estimating Euler equations and consumption growth moments, I can map preference parameters directly into prices of risk and adjust standard errors for parameter uncertainty in consumption growth dynamics. Second, in terms of inference, the GMM objective function for the disappointment model is not differentiable, and therefore consistency and asymptotic normality of the GMM estimates are not guaranteed. However, I show that the disappointment model satisfies the conditions for consistency of non-differentiable GMM estimators[8] because I assume that consumption growth is a continuous random variable. In contrast, these conditions do not hold when consumption growth takes discrete values (e.g., Bonomo et al. (2011)).

Finally, I extend the literature on reference-dependent preferences by showing that the location of the reference outcome affects the empirical performance of these models. Instead of calibrating a

[8]See Andrews (1994) and Newey and McFadden (1994).

single reference-based model, my estimation approach allows the data to decide on the significance of alternative reference points. Specifically, this is the first paper to show that, when the reference outcome is based on either the status quo assumption of Kahneman and Tversky (1979), the expectation model of Kőszegi and Rabin (2006), or a quickly adapting reference point as implied by Arkes et al. (2008), then downside consumption risk is not priced. In this case, the reference-dependent model is equivalent to the traditional consumption-based framework with symmetric preferences, and cannot explain the cross-section of expected returns.

2. Recursive utility with disappointment aversion

In this section, I introduce the generalized disappointment aversion discount factor, and obtain explicit solutions in terms of aggregate consumption growth.

2.1 The Generalized disappointment aversion stochastic discount factor

Consider a discrete-time, single-good, closed, endowment economy in which there is no productive activity. At each point in time, the endowment of the economy is generated exogenously by "tree-assets" (Lucas 1978). Equity claims for these "tree-assets" are traded in complete markets free of transaction costs. Disappointment averse investors are fully rational, face no restrictions on asset holdings, and are characterized by identical homothetic preferences.

Under these assumptions, there exists a representative investor (Routledge and Zin (2010)) who chooses consumption C_t and asset weights $\{w_{i,t}\}_{i=1}^n$ to maximize her lifetime utility V_t

$$V_t = \max_{C_t,\ \{w_{i,t}\}_{i=1}^n} \left[(1-\beta)C_t^\rho + \beta \mu_t(V_{t+1};\ V_{t+1} < \delta\mu_t)^\rho\right]^{\frac{1}{\rho}} \tag{1}$$

$$\text{with } \mu_t(V_{t+1};\ V_{t+1} < \delta\mu_t) = \mathbb{E}_t\Big[\frac{V_{t+1}^{-\alpha}\ (1+\theta\mathbf{1}\{V_{t+1}\ \leq\ \delta\mu_t\})}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1}\ \leq\ \delta\mu_t\}]}\Big]^{-\frac{1}{\alpha}}, \tag{2}$$

subject to $1 > \theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}$ and the usual budget and transversality constraints. Lifetime utility is strictly increasing in wealth, globally concave, linear homogeneous, and time-consistent, since V_t is increasing in V_{t+1}.[9]

The operator μ_t is the generalized disappointment aversion certainty equivalent, and $\mathbb{E}_t$ is the conditional expectation operator. The term $\mathbf{1}\{V_{t+1} < \delta\mu_t\}$ is the disappointment indicator that overweights bad states of the world (disappointment events) and shows when investors feel disappointed. The denominator in equation (2) is a normalization constant such that $\mu_t(\mu_t) = \mu_t$.

The key parameter in the disappointment model is the disappointment aversion coefficient $\theta \geq 0$. This parameter measures the asymmetry in investor preferences over gains and losses. If θ is positive, a \$1 loss in consumption during disappointment periods hurts approximately $1 + \theta$ times more than a \$1 loss in consumption during normal times. When θ is zero, investors have symmetric preferences, and the effects of first-order risk aversion vanish.

The constant $\delta > 0$ is the generalized disappointment aversion (GDA) parameter introduced in Routlegde and Zin (2010). This parameter is associated with the threshold for disappointment. According to (2), disappointment events happen whenever lifetime utility V_{t+1} is less than some multiple δ of its certainty equivalent μ_t. In Gul (1991), δ is 1, and disappointment events occur whenever utility falls below its certainty equivalent. On the other hand, according to the GDA framework, disappointment events may happen below or above the certainty equivalent, depending on whether the GDA coefficient δ is lower or greater than 1, respectively.

The constant $\alpha \geq -1$ is the traditional coefficient of second-order risk aversion which affects the piece-wise curvature of the utility function. The parameter $\beta \in (0, 1)$ is the discount rate. In the deterministic steady-state of the economy, an additional \$1 of consumption tomorrow is worth \$$\beta$ today. Finally, the EIS between two consecutive periods is given by $\frac{1}{1-\rho}$. The EIS measures the responsiveness of consumption growth to changes in the real interest rate. The magnitude of the EIS has important implications for asset pricing models. In Bansal and Yaron (2004), ρ is positive and the EIS is greater than 1. In contrast, Hall (1988) and Vissing-Jorgensen (2002) find that ρ is negative, and that the EIS is less than 1.

[9] In relation to alternative reference-dependent models, the disappointment aversion does not violate first-order stochastic dominance or transitivity of preferences (Gul (1991)).

Based on the recursion of equation (1), the GDA stochastic discount factor is given by[10]

$$M^{GDA}_{t,t+1} = \beta\Big(\frac{C_{t+1}}{C_t}\Big)^{\rho-1}\Big[\frac{V_{t+1}}{\mu_t\big(V_{t+1}; V_{t+1} < \delta\mu_t\big)}\Big]^{-\alpha-\rho}\Big[\frac{1+\theta\mathbf{1}\{V_{t+1} \ < \ \delta\mu_t\}}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta > 1\} + \theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{V_{t+1} < \delta\mu_t\}]}\Big]. \tag{3}$$

$M^{GDA}_{t,t+1}$ adjusts expected values by taking into account investor preferences over the timing and risk of stochastic payoffs. The first term in equation (3) corrects for the timing of uncertain payoffs which occur at a future date (resolution of uncertainty). The second term adjusts future stochastic payoffs for investors' dislike of risk (second-order risk aversion). When investor preferences are time-additive ($\alpha = -\rho$), adjustments for time and risk are identical and the second term is identically equal to 1.

The third term in equation (3) is the novel term in the stochastic discount factor due to disappointment aversion. This term adjusts stochastic payoffs for investors' aversion to disappointment. The GDA term distorts probability weights by shifting more mass to disappointment events, namely states of the world in which lifetime utility V_{t+1} is less than some multiple δ of its certainty equivalent μ_t. If investors are disappointment neutral ($\theta = 0$), the disappointment term is identically equal to 1.

According to the expression in (3), the GDA discount factor is a function of the observable consumption growth and the unobservable lifetime utility because household preferences are not separable across time. Epstein and Zin (1989) and Routledge and Zin (2010) show that these lifetime utility terms can be replaced by returns on aggregate wealth. In the next section, I use the methodology in Bansal and Yaron (2004) and the price-dividend log-linearization of Campbell and Shiller (1988) to express returns on aggregate wealth in terms of aggregate consumption growth.

2.2 Explicit solutions for the GDA stochastic discount factor

To solve the GDA discount factor in terms of consumption growth, I assume that consumption growth follows an autoregressive process (AR(1)) with constant volatility and i.i.d. $N(0, 1)$ shocks

[10] Hansen et al. (2007), Routledge and Zin (2010), and Appendix A here.

$$\Delta c_{t+1} = \mu_c(1-\phi_c) + \phi_c \Delta c_t + \sqrt{1-\phi_c^2}\sigma_c \epsilon_{c,t+1}. \tag{4}$$

The parameters μ_c, σ_c^2, and ϕ_c are the unconditional mean, variance, and first-order autocorrelation for consumption growth, respectively.[11] Even though the AR(1) process is quite standard in the asset pricing literature (e.g., Mehra and Prescott (1985), Routledge and Zin (2010)), I verify whether this assumption is valid while estimating the disappointment model.

Recent results in asset pricing (e.g., Bansal and Yaron (2004), Bonomo et al. (2011)) rely on unobservable persistent state variables or time-varying volatility to characterize the consumption growth process. Although persistent state variables and time-varying volatility are plausible assumptions, these assumptions are hard to detect empirically, at least at the annual frequency, due to limited time-series observations. For the same reason, it is difficult to jointly estimate Euler equations and a heteroscedastic model for consumption growth. Therefore, I consider a homoscedastic process for consumption growth as in Routledge and Zin (2010) to facilitate the joint estimation of expected returns and consumption growth moments. Next, I use the AR(1) process for consumption growth to derive explicit solutions for the GDA discount factor.

2.2.1 Explicit solutions for the wealth-consumption ratio

Based on my assumptions for consumption growth, I can obtain an empirically tractable version of the GDA stochastic discount factor in which unobservable lifetime utility is replaced by observable consumption growth.

Proposition 1: Given the AR(1) assumption for consumption growth in equation (4), the log price-dividend ratio for the claim on aggregate consumption ($p_{c,t} = log\frac{P_{c,t}}{C_t}$) is affine in consumption

[11] For $\phi_c = 0$, the AR(1) models nests the i.i.d. case. The AR(1) framework in (4) can easily be extended to vector auto-regressive processes.

growth $p_{c,t} = \mu_v + \phi_v \Delta c_t$ with

$$\mu_v = \frac{1}{1-\kappa_{c,1}}\Big[log\beta + \kappa_{c,0} + \frac{\rho(1-\phi_c)}{1-\kappa_{c,1}\phi_c}\mu_c + \frac{d_1\rho}{1-\kappa_{c,1}\phi_c}\sqrt{1-\phi_c^2}\sigma_c\Big], \quad \phi_v = \frac{\rho\phi_c}{1-\kappa_{c,1}\phi_c},$$

and d_1 is the solution to the fixed point problem:

$$d_1 = -\frac{\alpha}{2(1-\kappa_{c,1}\phi_c)}\sqrt{1-\phi_c^2}\sigma_c - \frac{log\Big[\frac{1+\theta N\left(d_1+\frac{(1-\kappa_{c,1}\phi_c)log\delta}{\sqrt{1-\phi_c^2}\sigma_c}+\frac{\alpha}{1-\kappa_{c,1}\phi_c}\sqrt{1-\phi_c^2}\sigma_c\right)}{1-\theta(\delta^{-\alpha}-1)+\theta N\left(d_1+\frac{(1-\kappa_{c,1}\phi_c)log\delta}{\sqrt{1-\phi_c^2}\sigma_c}\right)}\Big]}{\frac{\alpha}{1-\kappa_{c,1}\phi_c}\sqrt{1-\phi_c^2}\sigma_c}, \tag{5}$$

where $\kappa_{c,0}$, $\kappa_{c,1} \in (0,1)$ are linearization constants for returns on aggregate wealth and $N(.)$ is the standard normal c.d.f.

Proof: *See Appendix A.*

The parameter μ_v is the constant term in the log price-dividend ratio. It depends on the rate of time preference (β) and the unconditional mean for consumption growth (μ_c) adjusted for persistence $\left(\frac{\rho(1-\phi_c)}{1-\kappa_{c,1}\phi_c}\right)$ and disappointment $\left(\frac{d_1\rho}{1-\kappa_{c,1}\phi_c}\sqrt{1-\phi_c^2}\sigma_c\right)$. The constant ϕ_v is the sensitivity of the log price-dividend ratio to consumption growth, and its sign depends on the magnitude of the EIS. If the EIS is greater than 1, ϕ_v is positive, and the log price-dividend ratio for a claim on aggregate consumption is pro-cyclical. In contrast, if the EIS is less than 1, the log price-dividend ratio for a claim on aggregate consumption is counter-cyclical.[12]

Finally, d_1 in equation (5) is the disappointment threshold for consumption growth shocks, and is the solution to a fixed-point problem. In Appendix A, I show that the solution to the fixed-point problem exists and is unique. Moreover, for reasonable values of α, θ, and δ, d_1 is negative. Essentially, the disappointment threshold d_1 shows how many standard deviations below the mean consumption growth must fall before investors experience disappointment. For instance, my estimation results suggest that d_1 is approximately -1. This value implies that disappointment events occur whenever consumption growth drops more than one standard deviation below its

[12]Even if the log price-dividend ratio for the claim on aggregate consumption is counter-cyclical, the log price-dividend ratio for the stock market or any other asset can still be pro-cyclical (Bansal and Yaron (2004), p. 1485).

conditional mean. An important difference between the disappointment model in this paper and the one in Routledge and Zin (2010) is that here, the GDA certainty equivalent does not depend on the EIS parameter ρ because *Proposition 1* expresses returns on wealth in terms of consumption growth.

An immediate implication of *Proposition 1* is that the GDA stochastic discount factor from (3) can be written as

$$M_{t,t+1}^{GDA} = \tag{6}$$
$$exp\Big[log\beta + (\rho - 1)\Delta c_{t+1} + \frac{\alpha + \rho}{1 - \kappa_{c,1}\phi_c}[\mu_c(1-\phi_c) + d_1\sqrt{1-\phi_c^2}\sigma_c - \Delta c_{t+1} + \phi_c \Delta c_t]\Big]$$
$$\times \frac{1 + \theta \mathbf{1}\{\Delta c_{t+1} \leq (1-\kappa_{c,1}\phi_c)log\delta + \mu_c(1-\phi_c) + \phi_c\Delta c_t + d_1\sqrt{1-\phi_c^2}\sigma_c\}}{1 - \theta(\delta^{-\alpha} - 1) + \theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{\Delta c_{t+1} \leq (1-\kappa_{c,1}\phi_c)log\delta + \mu_c(1-\phi_c) + \phi_c\Delta c_t + d_1\sqrt{1-\phi_c^2}\sigma_c\}]}.$$

The GDA discount factor in equation (6) corrects expected future payoffs for time, risk, and disappointment similar to the discount factor in equation (3). The crucial difference between the two expressions is that, in (6), unobservable lifetime utility is expressed in terms of observable consumption growth. For instance, the geometric innovation in lifetime utility $\big(V_{t+1}/\mu_t(V_{t+1})\big)$ in equation (3) is replaced by an arithmetic innovation in log-consumption growth. This arithmetic innovation is amplified by consumption growth persistence $\left(\frac{1}{1-\kappa_{c,1}\phi_c}\right)$ to capture the long-term aspect of lifetime utility.

2.2.2 Disappointment events in consumption growth

Another implication of *Proposition 1* is that disappointment events can be expressed in terms of consumption growth rather than lifetime utility. According to the expression in (6), the disappointment threshold is equal to the certainty equivalent for consumption growth adjusted for the GDA parameter δ

$$\Delta c_{t+1} \leq (1-\kappa_{c,1}\phi_c)log\delta + \mu_c(1-\phi_c) + \phi_c\Delta c_t + d_1\sqrt{1-\phi_c^2}\sigma_c. \tag{7}$$

For $\delta = 1$, the threshold for disappointment is exactly equal to the certainty equivalent for consumption growth as in Gul (1991)

$$\Delta c_{t+1} \leq \mu_c(1 - \phi_c) + \phi_c \Delta c_t + d_1 \sqrt{1 - \phi_c^2} \sigma_c. \tag{8}$$

The expressions in (7) and (8) imply that disappointment events occur whenever next period's consumption growth is less than some quantity that depends on current consumption growth. At first glance, this result looks similar to the habit model (e.g., Campbell and Cochrane (1999)). However, the threshold value for disappointment events, i.e, the certainty equivalent of consumption growth is forward-looking. Moreover, in Campbell and Cochrane's habit model, consumption never drops below its habit; otherwise, marginal utility becomes infinite. On the other hand, in the disappointment model, periods during which consumption growth falls below its certainty equivalent are particularly important for asset prices.

2.3 The risk-free rate

A general equilibrium model should be able to fit expected asset returns as well as key moments for the risk-free rate. Based on the explicit solutions for the GDA discount factor, I can express the one-period risk-free rate as a function of consumption growth

$$r_{f,t+1} = -log\beta + [\mu_c(1 - \phi_c) + \phi_c \Delta c_t](1 - \rho) - h_1(\sigma_c) - h_2(\sigma_c^2), \tag{9}$$

where h_1 and h_2 are real functions which capture the precautionary savings motive. Precautionary savings are affected by both risk and disappointment aversion. The traditional risk aversion term ($h_2(\sigma_c^2)$) depends on the variance of consumption growth, while the disappointment aversion term ($h_1(\sigma_c)$) depends on consumption growth volatility (σ_c) due to first-order risk aversion. I will use the equilibrium condition in (9) to identify the EIS parameter ρ.

3. Data and estimation methodology

In the previous section, I solved the GDA stochastic discount factor in terms of consumption growth. In this section, I describe the data and the statistical methodology used to estimate the disappointment model.

3.1 Data

For the empirical analysis, I use annual data because annual consumption and population measures are more accurate than quarterly or monthly estimates. My sample spans a period from December 31, 1933 to December 31, 2012. Personal consumption expenditures (PCE) and PCE index data are from the BEA. Per capita consumption expenditures are defined as services plus non-durables. Each component of aggregate consumption expenditures is deflated by its corresponding PCE price index (base year is 2009).

Population data are from the U.S. Census Bureau and recession dates are from the NBER. Asset returns, factor returns, and the risk-free rate are from Kenneth French's (whom I kindly thank) website. Stock returns and interest rates are all in real terms and have been adjusted for inflation by subtracting the growth rate of the PCE price index. Finally, I follow the "beginning-of-period" convention, as in Campbell (2003) and Yogo, (2006), because beginning-of-period consumption growth is better aligned with stock returns.

Table 1 shows summary statistics for consumption growth, the risk-free rate, and equity returns. According to these results, the average risk-free rate is almost zero, while both consumption growth and the risk-free rate exhibit some persistence. The average stock market return is equal to 7.17% and the stock market Sharpe ratio is 0.38. Finally, the value premium over the 1933-2012 period is 4.58% and the size premium is 2.14%.

3.2 Estimation methodology

The structural parameters to be estimated are the discount rate β, the EIS parameter ρ, the risk aversion coefficient α, the disappointment aversion parameter θ, and the GDA coefficient δ. The key feature of disappointment aversion is that the reference point for disappointment d_1 is endogenous. According to the expression in (5), d_1 is identified once preference parameters and consumption growth moments have been estimated.

Following Bansal et al. (2013), my analysis focuses on value-weighted portfolios sorted on size and book-to-market (B/M). Ever since Fama and French (1993, 1996) documented that size and B/M capture most of the cross-sectional variation in equity returns, much of the asset pricing literature has focused on explaining the size and value factors. Because I jointly estimate Euler equations and consumption growth moments with a limited time-series sample, I am forced to consider a small cross-section, such as the 2×3 size-B/M sort.[13] I also consider the stock market portfolio and the risk-free rate as part of my test assets. Empirical asset pricing tests typically focus on the cross-section of expected returns, ignoring the moment conditions for the mean and variance of the risk-free rate. Nevertheless, these conditions are important for identification and constitute an additional hurdle for asset pricing factors. For instance, Ahn et al. (2003) discuss the importance of including the risk-free rate as a test asset to fix the mean of the stochastic discount factor at a reasonable level.

3.2.1 GMM moment conditions

Estimation is conducted via GMM (Hansen and Singleton (1982)). The GMM conditions are the unconditional moments for consumption growth and the unconditional Euler equations:

[13] Estimation results for 25 size-B/M, 10 size, 10 B/M, 10 earning-to-price, equal-weighted portfolios, nominal returns, as well as results for the post-war period are available upon request.

$$\mathbb{E}[g(z_t, x)] = \begin{bmatrix} \mathbb{E}[\Delta c_{t+1}] - \mu_c \\ \left(\mathbb{E}[\Delta c_{t+1}^2] - \mu_c^2\right) - \sigma_c^2 \\ \left(\mathbb{E}[\Delta c_{t+1} \Delta c_t] - \mu_c^2\right) - \phi_c \sigma_c^2 \\ \left(\mathbb{E}[(logR_{f,t+1})^2] - \mathbb{E}[logR_{f,t+1}]^2\right) - (1-\rho)^2 \phi_c^2 \sigma_c^2 \\ \mathbb{E}\Big[M_{t,t+1}^{GDA} R_{f,t+1}\Big] - 1 \\ \mathbb{E}\big[M_{t,t+1}^{GDA}\big(R_{i,t+1} - R_{f,t+1}\big)\big] \quad \text{for} \quad i = 1, 2, ..., n-1 \end{bmatrix}, \tag{10}$$

The vector z_t is the vector of consumption growth and asset returns and x is the vector of model parameters . The derivation of the unconditional Euler equations is discussed in Appendix A. I consider unconditional moments for two reasons. First, I have limited time-series observations. Second, given the homoscedasticity assumption for consumption growth the model is not really designed to produce realistic time-variation in conditional moments.

For my empirical tests, I use the first-stage GMM because, according to Cochrane (2001) and Liu et al. (2009), the first-stage GMM preserves the economic structure of the GMM objective function. Hayashi (2000, p. 229) also discusses the small sample properties of GMM estimators and suggests the use of first-stage GMM in small samples.

The structural parameters are estimated by minimizing the sample analog of the GMM objective:

$$\min_{\{x\}} \hat{\mathbb{E}}[g(z_t, x)]' \; W \; \hat{\mathbb{E}}[g(z_t, x)]. \tag{11}$$

Moment conditions are weighted by a prespecified diagonal matrix W whose first six diagonal elements are large numbers (10^8) and the remaining elements are equal to one. This is done for two reasons. First, moment conditions have different scales because the GMM system includes both consumption growth and asset pricing moments. Second, overweighting consumption moments ensures that the various asset pricing models are not fitting expected returns by inflating the variability or the persistence of the consumption growth process. Essentially, the suggested weighting scheme tests whether the various asset pricing models can explain the size and B/M portfolios while perfectly fitting key moments for consumption growth, the risk-free rate, and the stock market.

Finally, using the definition of covariance and the unconditional Euler equations from the expression in (10), I can generate fitted expected returns $\hat{\hat{\mathbb{E}}}[R_{i,t}]$, as follows:

$$\hat{\hat{\mathbb{E}}}[R_{i,t+1}] = \hat{\mathbb{E}}[R_{f,t+1}] - \frac{1}{\hat{\mathbb{E}}[M_{t,t+1}]}\widehat{\mathbf{Cov}}[R_{i,t+1} - R_{f,t+1}, M_{t,t+1}]. \tag{12}$$

To assess the asset pricing performance of the various consumption models, I employ several measures of fit: the cross-sectional R^2, the cross-sectional root mean square error ($RMSE$), and the J-test (Hansen (1982)).[14] The cross-sectional R^2s and the $RMSE$ are calculated for the market and the six size-B/M portfolios to maintain consistency across models for which consumption growth moments or the volatility of the risk-free rate are not part of the GMM moment conditions.

3.2.2 Identification

According to the solution for the GDA discount factor in equation (6), joint identification of the risk aversion and EIS parameters is difficult. However, based on the expression for the risk-free rate in equation (9), the variance of the risk-free rate depends only on the variance of consumption growth through the inverse of the EIS, since

$$Var(r_{f,t+1}) = (1-\rho)^2\phi_c^2 Var(\Delta c_t). \tag{13}$$

This condition allows me to identify ρ using the variance of the risk-free rate and my assumption that consumption growth is a conditionally homoscedastic AR(1) process. Even though the expression in equation (13) is quadratic in ρ, I can use the restriction $\rho < 1$ to identify unique solutions. For

[14]The expressions for the cross-sectional R^2 and the $RMSE$ are given by

$$R^2 = 1 - \frac{\sum_{i=1}^N(\hat{\hat{\mathbb{E}}}[R_{i,t}] - \hat{\mathbb{E}}[R_{i,t}])^2}{\sum_{i=1}^N(\hat{\mathbb{E}}[R_{i,t}] - \frac{1}{N}\sum_{i=1}^N \hat{\mathbb{E}}[R_{i,t}])^2} \text{ and } RMSE = \sqrt{\frac{1}{N}\hat{\mathbb{E}}[g(z_t,x)]'\hat{\mathbb{E}}[g(z_t,x)]}, \text{ respectively.}$$

$\hat{\mathbb{E}}[R_{i,t}]$ are sample expected returns, and $\hat{\mathbb{E}}[g(x_t,x)]$ is the vector of average pricing errors from equation (11).

instance, $Var(r_{f,t+1})/\big(\phi_c^2 Var(\Delta c_t)\big)$ is approximately 36 in the data, which means that ρ is either -5 or 7. However, the positive solution cannot be accepted because ρ has to be lower than 1.

Delikouras (2014b) and Dolmas (2014) show that joint identification of the risk and disappointment aversion parameters is difficult when the equity premium is the only test asset. To address this concern, I follow Ostrvonaya et al. (2006) and first estimate the disappointment model for a set of prespecified risk aversion parameters ($\alpha \in \{-1, 1, 3\}$). The first set of results suggests that joint estimation of θ, and α is feasible due to the inclusion of the risk-free asset and the six size-B/M portfolios in the set of test assets. Therefore, in a second set of tests, the risk aversion coefficient is jointly estimated with the rest of the parameters. Finally, since the price for a claim on aggregate consumption is unobservable, I assume that the linearization constant $\kappa_{c,1}$ from equation (5) is equal to the rate of time preference β. This assumption is true for log-time preferences when ρ is zero and the EIS is equal to 1.

3.2.3 Consistency and asymptotic normality

Establishing consistency and asymptotic normality for the disappointment model is challenging because the GDA discount factor in equation (6) is not continuous and the GMM objective functions is not differentiable. However, Andrews (1994) and Newey and McFadden (1994) show that, for non-continuous estimators, continuity and differentiability of the GMM objective function can be replaced by the less stringent conditions of continuity with probability 1 and stochastic differentiability.[15] As shown in Appendix B, both of these conditions are satisfied by the GDA stochastic discount factor because I assume that consumption growth and stock returns are continuous random variables with a well-defined moment generating function. In contrast, the consistency arguments in Newey and McFadden for non-differentiable GMM estimators might not hold for disappointment models in which the state-space for consumption growth is discrete (e.g., Bonomo et al. (2011)).

Finally, confidence intervals and test statistics have been estimated using Kunsch's (1989) block bootstrap method, which is described in Appendix C. This method addresses some practical issues

[15]Theorems 2.6, 7.2, and 7.3 in Newey and McFadden (1994).

with non-continuous GMM estimation that are discussed in Appendix B. Hypothesis testing is conducted based on 95% confidence intervals instead of standard errors or t-statistics because the bootstrapped distribution of the estimated parameters is not symmetric. Next, I use the GMM framework to estimate the disappointment model.

4. GMM results for the consumption-based GDA discount factor

For the first set of empirical tests, I estimate the GDA discount factor from equation (6) using the consumption growth moments, the mean and volatility of the risk-free rate, and the Euler equations for the stock market and the six size-B/M portfolios. I also estimate two alternative consumption models: the Epstein-Zin (Epstein and Zin (1989)) and the CRRA specifications (Mehra and Prescott (1985)). These models are nested by the GDA model. For example, investors in the Epstein-Zin model are disappointment neutral ($\theta = 0$), while, in the CRRA model, investors are disappointment neutral and the risk aversion parameter is equal to the inverse of the EIS ($\alpha = -\rho$).

Following Ostrovnaya et al. (2006), I first estimate five different versions of the disappointment model (GDA(1) - GDA(5)) based on prespecified values for the risk aversion parameter ($\alpha \in \{-1, 1, 3\}$). This is done to address possible identification concerns regarding the joint estimation of the risk and disappointment aversion coefficients. For the first GDA specification (GDA(1)), I set α equal to -1 and assume $\alpha = -\rho$ to test the performance of a disappointment model in which investors are disappointment averse but risk neutral. For this specification, the EIS is infinite and the fitted volatility of the risk-free rate is zero. Finally, for the first two GDA specifications (GDA(1) and GDA(2)), I set the threshold for disappointment is exactly equal to the certainty equivalent ($\delta = 1$) as in Gul (1991). The results for the first set of tests are shown in Table 2.

4.1 Risk preferences

The key parameter in the GDA discount factor is the disappointment aversion coefficient θ. The estimates of θ in Table 2 are positive, with values ranging from 7.608 to 9.970. According to the expression for the GDA discount factor in equation (6), when θ is equal to 7.608, investors penalize losses during disappointment events 8.608 times more than losses during normal times. Moreover, based on the 95% confidence intervals, I cannot reject the hypothesis that the aggregate investor is disappointment averse, since the estimates of disappointment aversion coefficient are statistically significant across all GDA models. Finally, the results in Table 2 document a substitution effect between disappointment aversion and risk aversion. For example, the disappointment aversion estimates decrease monotonically from 9.970 in GDA(3) to 7.608 in GDA(5) as the risk-aversion parameter increases from -1 to 3, respectively.

The results for the GDA models are consistent with Routledge and Zin (2010) who set θ equal to 9. However, the θ estimates in Table 2 are not consistent with experimental evidence at the micro level. For instance, in portfolio choice problems, Choi et al. (2007) find disappointment aversion coefficients that range from 0 to 1.876 with a mean of 0.390. Similarly, using experimental data on real effort provision, Gill and Prowse (2012) estimate disappointment aversion coefficients that range from 1.260 to 2.070.

The different estimates of the disappointment aversion parameter at the micro and macro levels can be explained by the fact that, at the aggregate level, the disappointment aversion parameter probably depends on the homoscedasticity assumption for consumption growth. Specifically, disappointment models that assume heteroscedastic consumption growth are able to match asset pricing moments using reasonable magnitudes for the disappointment aversion parameter. For example, Bonomo et al. (2011) consider a stochastic variance process for consumption growth, and set θ equal to 2.33. Delikouras (2014b) also uses time-varying volatility for consumption growth and fits asset prices with a disappointment aversion parameter equal to 1.439. However, I cannot jointly estimate Euler equations and a heteroscedastic model for consumption growth using annual data. Therefore, in this paper, I follow Routledge and Zin (2010) and assume constant volatility for consumption growth to facilitate the empirical analysis.

An important feature of the disappointment model is that reference points for gains and losses are endogenous and are based on the certainty equivalent of consumption growth. For the GDA models (1) and (2) in Table 2, the GDA parameter δ is set equal to 1 as in Gul (1991). In contrast, δ is unrestricted for the GDA models (3) - (5) as in Routledge and Zin (2010). According to the results in Table 2, my empirical setting cannot distinguish between Gul's disappointment model and Routledge and Zin's generalized disappointment theory. Specifically, for the GDA models (3) and (4), the point estimate of δ is 0.997 whereas for the GDA(5) model the GDA estimate is 1.000. Moreover, based on the confidence intervals in Table 2, it is difficult to rule out the hypothesis that δ is equal to one.

In addition to the GDA coefficient δ, the disappointment threshold is also affected by the parameter d_1, which, according to equation (5), is a function of preference parameters and consumption growth moments. The estimates for d_1 in Table 2 range from -0.923 to -0.789. These results imply that, on average, investors feel disappointed whenever consumption growth is less than 0.8 standard deviations below the mean. The results in Table 2 also show that the disappointment threshold and the probability of disappointment events increase monotonically as the estimates of the disappointment aversion coefficient decrease. For instance, in the GDA(3) specification, θ is 9.970 and the probability of disappointment events is 10%, whereas in the GDA(5) model, θ is 7.608 and the probability of disappointment events increases to 14%.

Overall, according to the results in Table 2, the estimates of the disappointment aversion coefficient are economically and statistically significant across all the GDA specifications. However, the magnitude of these estimates depends on the prespecified values for the risk aversion parameter. Next, I discuss the estimates of the EIS and the discount rate.

4.2 Time preferences and consumption growth moments

An important question in the consumption-based asset pricing literature is whether the EIS is lower or greater than 1. Estimating the EIS in models with non-separable preferences is not trivial because, according to the expression in (6), α and ρ cannot be separately identified. However, if consumption growth is predictable and homoscedastic, then the EIS can be identified using the moment condition for the variance of the risk-free rate in equation (13). The estimates of ρ in

Table 2 are negative, with values ranging from -8.280 to -5.122, and are statistically significant across all the GDA specifications. Based on the estimates for ρ, the EIS ranges from 0.10 to 0.16. These values imply that, when the risk-free rate increases by 1%, consumption growth increases by approximately 0.10% to 0.16%. These results are consistent with the findings in Hall (1988), Vissing-Jorgensen (2002), and Yogo (2004b), who report that the EIS is much less than 1. In contrast, Bansal and Yaron (2004) and Bansal, Dittmar, and Lundblad (2005) assume that ρ is positive and that the EIS is greater than 1.

The estimate for the discount rate β in the GDA(1) model is 0.998. This means that, in the deterministic steady-state of the economy, \$1 of consumption tomorrow is worth \$0.998 today. The estimates of β for the rest of the GDA specifications (GDA(2) - GDA(5)) are greater than 1 (β = 1.067 - 1.092). However, according to the discussion in Piazzesi and Schneider (2006), discount rates in non-separable models can be greater than 1.

Finally, Table 2 shows estimation results for the consumption growth moments. Expected consumption growth is 2.169%, consumption growth volatility is 1.516%, and the autocorrelation coefficient for consumption growth is approximately 0.300. These results imply that consumption growth exhibits some persistence and is probably not an i.i.d. process. These consumption growth moments are almost identical across all GDA specifications, and are all statistically significant at the 5% level. Further, the GMM estimates for consumption growth moments are very similar to the stand-alone calculations from Table 1. For example, in Table 1, expected consumption growth is 2.167%, consumption growth volatility is 1.556%, and the autocorrelation coefficient is 0.371. Next, I discuss results for the Epstein-Zin and CRRA models.

4.3 GMM results for the CRRA and Epstein-Zin models

Table 2 shows estimation results for the CRRA and Epstein-Zin models in which investors are disappointment neutral. In the CRRA model, the risk aversion estimate is 7.218 and the EIS is $1/(1 + 7.218) = 0.121$ since, in this model, the EIS is equal to the inverse of the risk aversion coefficient. Nevertheless, based on the 95% confidence interval, I cannot reject the hypothesis that the risk aversion parameter for the CRRA model is statistically equal to zero. Contrary to the CRRA specification, in the Epstein-Zin model, preferences over risk and time are characterized by different parameters. Consistent with the results in Routledge and Zin (2010), the risk aversion coefficient in the Epstein-Zin model is statistically significant and equal to 38.369. This estimate is four times greater than the risk aversion parameter in the long-run risk framework of Bansal and Yaron (2004).[16]

To explain the difference between the risk aversion estimate for the Epstein-Zin model in Table 2 and the one in the long-run risk model of Bansal and Yaron (2004), consider the following approximation for fitted expected log-returns according to the Epstein-Zin model[17]

$$\mathbb{E}[R_{i,t+1} - R_{f,t+1}] \approx -\mathbf{Cov}\big(e^{-(\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}+1-\rho)\Delta c_{t+1}}, R_{i,t+1} - R_{f,t+1}\big). \tag{14}$$

If the persistence in consumption growth ϕ_c is high enough that $1 - \kappa_{c,1}\phi_c \approx 0$, the Epstein-Zin model in equation (14) can generate large equity risk premiums, even if the coefficient of risk aversion α is low. If, additionally, ρ is positive, the effects of the consumption growth persistence are even more pronounced.[18] For instance, based on the calibrated parameters from Bansal and Yaron, the effective risk aversion coefficient in equation (14), after taking into account the persistence in consumption growth, is equal to 390.443.[19] In contrast, for low persistence in consumption growth and negative ρ estimates, the Epstein-Zin model requires a large risk aversion coefficient, around 40, to explain the equity premium. Next, I discuss the fit of the consumption-based disappointment models.

[16] Bansal and Yaron (2004) set the risk aversion parameter for the long-run risk model is equal to 9.

[17] This approximation is based on equations (6) and (12) and the empirical facts that $\mathbb{E}[M_{t+1}] \approx 1$, and $\widehat{\mathbf{Cov}}(r_{i,t+1} - r_{f,t+1}, \Delta c_t) \approx 0$.

[18] Beeler and Campbell (2012) suggest that the equity risk premium in the long-run risk model of Bansal and Yaron (2004) is generated by an extremely persistent consumption growth process and a high EIS. If the EIS is less than 1 or if consumption growth is i.i.d., equity premia in the long-run risk model are almost zero unless the risk aversion coefficient is large.

[19] In Bansal and Yaron (2004), $\alpha = 9$, $\rho = 0.333$, $\kappa_1 = 0.997$, and $\phi_c = 0.979$.

4.4 Pricing errors for the consumption-based GDA discount factor

To assess the asset pricing performance of the various consumption models, Table 2 shows several measures of fit: the cross-sectional R^2, the root mean square error ($RMSE$), and the J-test (Hansen (1982)).

The negative value R^2 for the CRRA model (R^2 = -11.235, $RMSE$ = 10.296%) suggests that time-additive preferences cannot simultaneously explain the low volatility of the risk-free rate and the cross-section of expected returns. This is because the variance of the risk-free rate in equation (13) constrains the magnitude of the EIS and, in the CRRA case, it also determines the value of the risk aversion coefficient. The fit of the consumption model is improved once we disentangle risk attitudes from time preferences with the Epstein-Zin specification. In this case, the R^2 becomes 71% and the $RMSE$ is 1.593%. The fit of the consumption model improves even more once we account for disappointment aversion. The R^2 for the disappointment model ranges from 94.5% to 99%, and the corresponding $RMSE$ ranges from 0.281% to 0.688%.

Even though the R^2 for the Epstein-Zin model is large, this model is rejected by the J-test. This finding is consistent with Lewellen et al. (2010), who argue that cross-sectional R^2s are typically large and cannot discern between asset pricing models. In contrast, disappointment models are the only consumption-based models not rejected by the J-test, with p-values ranging from 0.422 for the GDA(5) model to 2.291 for the GDA(1) specification. The GDA(1) model is rejected by the J-test (J-test = 51.267), even though it can almost perfectly fit expected returns (R-square = 95%), because it cannot fit the variance of the risk-free rate since, in this model, the inverse of the EIS is restricted to zero.

An important issue with the J-test is that it might fail to reject the null hypothesis because the denominator in the J-statistic is large (the model is not accurate), and not because the numerator in the J-statistic is small (the average pricing error is zero). To examine whether disappointment aversion truly improves the fit of consumption models, Table 3 shows average pricing errors (alphas) for the six size-B/M portfolios across all consumption models. The pricing errors for consumption growth moments, the risk-free rate, and the equity premium are omitted from Table 3 because these moments are perfectly fitted by all models, with the exception of the CRRA specification.

According to the results in Table 3, the pricing errors for the CRRA utility are large and statistically significant across all portfolios. Even though the Epstein-Zin model performs much better than the CRRA specification, Epstein-Zin preferences can only price two out of the six size-B/M portfolios. For example, the pricing errors of the Epstein-Zin model for the small-growth, the small-value, and the big-value portfolios are relatively large and statistically significant. In contrast, the pricing errors of the GDA models are practically zero across all the specifications. On average, the pricing errors for the GDA models are three times lower than the errors for the Epstein-Zin specification.

The results in Table 3 are also confirmed in Figure 1, which shows fitted and sample expected returns for the risk-free asset, the stock market, and the six size-B/M portfolios. According to Figure 1, with the exception of the CRRA specification, all models can perfectly fit the mean and volatility of the risk-free rate as well as the equity premium. In contrast, the CRRA model can either explain the volatility of the risk-free rate or the cross-section of expected stock returns, but not both. Although the Epstein-Zin discount factor is a major improvement over the CRRA model, its cross-sectional fit is far from perfect, especially for the small-value and small-growth portfolios. However, once I introduce disappointment aversion, fitted moments are perfectly aligned with sample ones across all assets. According to Figure 1, the disappointment model can explain the equity, size, and value premiums, as well as the mean and volatility of the risk-free rate.

Taken together, the results in Figure 1 and Table 3 suggest that disappointment risk can almost perfectly fit the cross-section of expected returns. Interestingly, a single-factor consumption-based model in which investors are disappointment averse but risk-neutral (the GDA(1) model) can explain 95% of the cross-sectional variation in expected returns. Moreover, the almost zero pricing errors for the GDA discount factor in Table 3 indicate that the disappointment model can fit both the cross-sectional variation and the level of expected stock returns.

4.5 Willingness-to-pay calculations

The effective risk aversion of disappointment averse investors depends on the risk and disappointment aversion coefficients. To assess the effective risk aversion implied by the estimates in Table 2, Panel A in Table 4 shows static willingness-to-pay calculations as in Epstein and Zin (2001). The numbers in Table 4 show the amount of money an investor would pay to avoid a gamble that pays $\$100,000 + \epsilon$ in which ϵ is a $N(0, \sigma_\epsilon)$ random variable. These numbers are essentially the difference between the mean and the certainty equivalent of the gamble for different values of σ_ϵ. Large numbers in Table 4 denote very risk averse behavior.

According to the findings in Table 4, for small gambles, disappointment averse investors are more risk averse than Epstein-Zin individuals due to the first-order risk aversion effect. However, for medium and large gambles, the GDA models of Table 2 imply a much less risk averse behavior than the Epstein-Zin model.

The previous calculations assume that preferences are separable over time. However, in a dynamic setting with non-separable preferences, the effective risk aversion is also affected by the EIS and the persistence of consumption growth. Therefore, in Panel B of Table 4, I show willingness-to-pay calculations in which the effective risk aversion parameter ($\tilde{\alpha}$) takes into account the persistence of consumption growth and the EIS. In this case, the effective risk aversion parameter is given by $\tilde{\alpha} = \left(\frac{\alpha+\rho}{1-\kappa_1\phi_c} - \rho\right)$ as in the expression (6). Based on the evidence from Panel B, for reasonable values of consumption growth persistence and a negative EIS, the willingness-to-pay of disappointment averse investors with non-separable preferences is quite similar to the case with separable preferences.

Overall, the results in Table 4 suggest that the risk and disappointment estimates in Table 2 are quite large. Nevertheless, based on the results from Table 3, the traditional consumption-based framework cannot explain expected returns even if the risk aversion coefficient is large. In contrast, the consumption-based GDA model with a large disappointment aversion coefficient can almost perfectly fit key moments for the risk-free rate, the stock market, and the six size-B/M portfolios.

Collectively, the first set of empirical results show that the estimates for the disappointment aversion coefficient are positive and statistically significant and that the EIS is less than 1. However, these results are inconclusive about the magnitude of the GDA parameter δ. Finally, even though the magnitude of the disappointment aversion parameter for the aggregate investor is not consistent with the evidence at the micro level, the GDA model can perfectly fit asset prices and it is the only consumption-based model not rejected by J-test. Next, I discuss the GDA mechanism in detail.

5. Disappointment events and the GDA mechanism

In this section, I identify the disappointment periods that are relevant for asset pricing and describe the GDA mechanism.

5.1 Disappointment events in consumption growth and NBER recessions

An important advantage to estimating the disappointment model is that I can identify actual disappointment events in consumption growth and examine how these events relate to aggregate macroeconomic conditions (e.g., recessions). According to the results in Table 2, the number of disappointment years is different across the various GDA specifications, because the threshold for disappointment depends on the estimates of the disappointment aversion coefficient.[20] Nevertheless, the number of disappointment events does not really affect the fit of the different GDA specifications, because all these models are able to identify a common set of disappointment years that are important for asset prices.

To study the relation between disappointment events and the macroeconomy, Figure 2 plots consumption growth, disappointment events, and NBER recession dates. Disappointment events are highlighted by ellipses, and are estimated for the GDA(5) model in Table 2. According to Figure 2, disappointment events are linked to real economic activity since these events usually happen right before or during a recession. Disappointment events do not always overlap recessions

[20]The disappointment years for the GDA(5) model are: 1937, 1946, 1948, 1953, 1956, 1973, 1979, 1990, 1999, 2007, and 2008. The set of disappointment years for the rest of the GDA models excludes 1948, 1953, and 1956.

because NBER recessions and disappointment events are defined differently. For example, according to an NBER report (2003):

> *A recession is a significant decline in economic activity spread across the economy, lasting more than a few months, normally visible in real GDP, real income, employment, industrial production, and wholesale-retail sales.*

In contrast, according to the expression in (7), disappointment events are related to periods of much worse-than-expected economic activity rather than to periods of low economic activity. In some cases, the two events coincide (e.g., 1937, 1953, 1973, 1990, 2008) but in others, disappointment events tend to occur before recessions (e.g., 1946, 1948, 1956, 1979, 1999, 2007).

Even if disappointment periods do not always overlap recessions, it seems that investors are quite sensitive to disappointment events and demand high risk premiums for holding assets that perform poorly during these events. For instance, the equity premium is large because the stock market exposes the aggregate investor to disappointment risk. Similarly, value (small) firms command high risk premiums because stock returns of these firms covary more, in absolute value, with disappointment events than stock returns of growth (big) firms do. Next, I examine the disappointment mechanism in more detail.

5.2 Disappointment risk and the GDA mechanism

This section sheds additional light on the GDA mechanism by considering the following approximation for fitted expected returns according to the disappointment model:

$$\mathbb{E}[R_{i,t+1} - R_{f,t+1}] \approx -\mathbf{Cov}\big(e^{-(\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}+1-\rho)\Delta c_{t+1}}, R_{i,t+1} - R_{f,t+1}\big) \tag{15}$$
$$-\theta\mathbf{Cov}\big(\mathbf{1}\{\Delta c_{t+1} \leq (1-\kappa_{c,1}\phi_c)log\delta + \mu_t(\Delta c_{t+1})\}e^{(\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}+1-\rho)\Delta c_{t+1}}, R_{i,t+1} - R_{f,t+1}\big).$$

This expression shows that stocks compensate investors for two sources of systematic risk.[21] The first is consumption growth risk, and it is captured by the covariance between asset returns and consumption growth. Consumption growth is multiplied by the constant $\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}+1-\rho$, which is the price of consumption risk. Because the model is non-separable, the price of consumption risk depends on the risk aversion parameter (α), the inverse of the EIS ($1-\rho$), and the persistence of consumption growth (ϕ_c). For the time-separable CRRA model, $\alpha=-\rho$ and, therefore, the price of consumption risk depends only on the inverse of the EIS ($1-\rho$), which is also the risk-aversion parameter.

The second source of systematic risk is disappointment risk. Disappointment risk is captured by the covariance between asset returns and the disappointment indicator. This covariance is scaled by the disappointment aversion parameter θ, which measures the asymmetry in investor preferences over gains and losses. Based on the expression in (15), θ is also the price of downside consumption risk (disappointment risk). Traditional consumption models assume that the price for disappointment risk is zero, and consider only the first covariance term in equation(15). Nevertheless, a large body of evidence documents that downside risk is priced both at the individual and at the aggregate levels.[22]

The results in Table 2 also confirm that disappointment risk is priced in the cross-section of expected returns. In fact, the almost zero pricing errors for the GDA(1) model, in which investors are disappointment averse but risk-neutral ($\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}+1-\rho=0$), imply that expected returns are almost exclusively compensation for disappointment risk. Taken together, the empirical results in Table 2 and Table 3 suggest that a single-factor consumption-based model with disappointment risk alone (GDA(1) model) can fit both the cross-sectional dispersion of expected returns as well as the level of the equity, value, and size premiums. This finding implies that expected returns are predominantly compensation for macroeconomic downside risk.

In addition to estimating the price of disappointment risk, this paper also shows that the

[21] This approximation is based on equations (6) and (12), and the empirical facts that $\mathbb{E}[M_{t+1}]\approx 1$, and $\widehat{\mathbf{Cov}}(R_{i,t+1}-R_{f,t+1},e^{\Delta c_t})\approx 0$.

[22] Epstein and Zin (1990), Choi et al. (2007), Routledge and Zin (2010), Gill and Prowse (2012).

significance of these estimates is affected by the assumptions regarding the location of the reference point (the kink in the utility function). Specifically, I find that downside consumption risk is not priced in reference-dependent models that consider alternative reference points for gains and losses. These results are discussed in the next section.

6. Alternative first-order risk aversion models

Reference-dependent models are characterized by their assumptions regarding the location of the reference point. Disappointment theory takes a strong stance on this issue: outcomes are evaluated with respect to their certainty equivalent. In this section, I compare the performance of the GDA framework to a set of first-order risk aversion models that specify alternative reference points for gains and losses.

The most famous member of the reference-dependent class of preferences is probably the loss aversion model of Kahneman and Tversky (1979).[23] Loss aversion shares a number of common features with disappointment aversion. For instance, the value function is defined on deviations from reference levels, and it is steeper for losses than for gains.[24] However, the original loss aversion framework and the majority of its subsequent empirical applications do not explain how reference points are formed or dynamically updated. Towards the end of their paper, Kahneman and Tversky discuss time-varying reference outcomes, but their entire analysis is based on the assumption that the reference point is the "status quo or one's current assets."

Contrary to Kahneman and Tversky, Kőszegi and Rabin (2006) suggest that investors do not necessarily define gains and losses with respect to the status quo. Instead, they propose that expectations are a more suitable benchmark for evaluating outcomes. Therefore, I also consider a

[23]In addition to loss aversion, probability weighting is another ingredient in Kahneman and Tversky's (1979) prospect theory. In this paper, I ignore probability weighting, and maintain the rational expectations assumption because my goal is to examine the explanatory power of asymmetric utility alone.

[24]In the loss aversion framework, the value function is concave over gains and convex for losses. A number of recent results (e.g., Duncan (2010)) question the S-shaped utility function. When the utility function is S-shaped, the second order necessary conditions for utility maximizaton must also be checked. In contrast, the disappointment aversion utility function is globally concave.

reference-based model in which the reference point is expected consumption growth. To calculate expected consumption growth, I maintain the assumptions of rational expectations and AR(1) consumption growth.

Finally, Arkes et al. (2008) show that reference points tend to adapt quickly to past experience, and that reference-point adaptation is greater following gains than following losses of equivalent size. Based on the discussion in Arkes et al., I set the reference point equal to last period's consumption growth in order to capture a quickly adapting reference point.[25] When consumption growth is an AR(1) process with positive autocorrelation, the GDA certainty equivalent in equation (7) is also adaptive. However, when the reference point is equal to last period's consumption growth, the reference point adjusts faster than in the GDA model in which the speed of adjustment is lower than 1 since $\phi_c < 1$. Finally, an alternative motivation for using last period's consumption growth as a reference point is a status quo model in growth rates.

Based on the previous discussion, the general first-order risk aversion stochastic discount factor (FORA sdf) is given by

$$M_{t,t+1}^{FORA} = exp\Big[log\beta + (\rho-1)\Delta c_{t+1} + \frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}\mu_c(1-\phi_c) - \frac{(\alpha+\rho)\alpha}{(1-\kappa_{c,1}\phi_c)^2}(1-\phi_c^2)\sigma_c^2 \tag{16}$$
$$-\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}\Delta c_{t+1} + \frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}\phi_c\Delta c_t\Big] \times \frac{1+\theta\mathbf{1}\{\Delta c_{t+1} \ \leq \ d\}}{1+\theta\mathbb{E}_t\big[\mathbf{1}\{\Delta c_{t+1} \leq d + \frac{\alpha}{1-\kappa_{c,1}\phi_c}(1-\phi_c^2)\sigma_c^2\}\big]}.$$

The constant d is the parameter that determines the location of the reference point. For $d = 0$, the reference level is zero consumption growth (the status quo) based on the loss aversion model of Kahneman and Tversky (1979). For $d = \mathbb{E}_t[\Delta c_{t+1}]$, the reference level is expected consumption based on the expectation model of Kőszegi and Rabin (2006). For $d = \Delta c_t$, the reference level is last period's consumption growth as implied by the quick adaptation evidence in Arkes et al. (2008). For comparison, I also estimate a model in which the location of the reference level is an unknown constant ($d = d_0$) to be estimated by GMM, and a specification in which the reference

[25] This framework does not allow for asymmetric adaptation of the reference point following gains and losses. To account for asymmetric effects in reference-point adaptation, in untabulated tests, I set the reference point equal to $\max\{\Delta c_t, 0\}$. These results are very similar to the ones shown here.

level is equal to the GDA certainty equivalent of consumption growth as in equation (6). Finally, unlike the results in Table 2, for this set of tests, the risk aversion coefficient α is jointly estimated with the rest of the parameters.

6.1 GMM results for the FORA discount factor

Table 5 shows estimation results for the alternative first-order risk aversion models. The most important finding in Table 5 is that the economic significance of the first-order risk aversion parameter θ depends on the assumption regarding the location of the reference point. For instance, when the reference level is located at zero (FORA(1) model), the estimate of the first-order risk aversion coefficient is statistically and economically insignificant while the estimate of the risk aversion parameter is around 39. When the reference level is equal to expected consumption growth (FORA(2) model), θ is also insignificant and α is 39. Similar results also hold for the quick adaptation model (FORA(3) model).

Overall, the estimates for the FORA models (1) - (3) are almost identical to the results for the Epstein-Zin model in Table 2 in which the first-order risk aversion parameter is zero by assumption. In contrast, when the location of the reference level is a constant parameter that is estimated by GMM (FORA(4) model), downside consumption risk is priced. In this case, the estimate of the first-order risk aversion parameter θ is 8.488, and the estimate of the loss threshold for consumption growth (d_0) is 1.097%. Both estimates are statistically significant at the 5% level. According to these estimates, whenever consumption growth is less than 1.097%, investors in the FORA(4) model penalize losses 9.488 times more than during normal times.

Finally, Table 5 shows results for the FORA model (FORA(5) model) in which the reference level is equal to the generalized certainty equivalent of consumption growth as in the GDA model. In Table 5, the risk aversion coefficient for the disappointment model is jointly estimated with the rest of the parameters whereas in Table 2, the risk aversion parameter for the disappointment models is fixed. When preference parameters for the disappointment model are jointly estimated, the disappointment aversion estimate is 7.232, the risk aversion estimate is 3.325, and both coefficients are significant at the 5% level. Finally, the estimate of the GDA parameter δ for the FORA(5) model is 0.996. However, based on the confidence interval for δ, we cannot rule out that δ is equal to one.[26]

The rest of the parameters are similar across the different FORA models and consistent with the findings in Table 2. For example, the EIS is around 0.10, the discount rate is greater than 1, and the AR(1) assumption for consumption growth is consistent with the data. Next, I examine the fit of the FORA models.[27]

6.2 Pricing errors for the FORA discount factor

The previous results show that the location of the reference point affects the statistical significance of the first-order risk aversion coefficient. In this section, I show that the location of the reference level also affects the empirical fit of the FORA models.

According to the results in Table 5, the cross-sectional fit of the status quo, expectation, and quick adaptation models (FORA(1), FORA(2), and FORA(3) models) is identical to the Epstein-Zin model from Table 2 ($R^2 = 70\%$, $RMSE = 1.594\%$). These models are rejected by the J-test (p-value $= 0$) because they cannot fit the cross-section of expected stock returns. The constant parameter specification (FORA(4) model) is also rejected by the J-test (J-test $= 21.413$, p-value $= 0$) and its cross-sectional fit is only a minor improvement over the first three FORA models ($RMSE = 1.436\%$) because consumption-based reference points are adaptive.

The only FORA model that is not rejected by the J-test in Table 5 is the GDA discount factor (FORA(5) model) with a J-statistic of 0.632 (p-value $= 0.959$). The fit of this model is quite impressive ($R^2 = 94\%$, $RMSE = 0.734\%$) implying that disappointment theory can successfully describe the reference level for gains and losses. For comparison, Table 5 also shows results for the Fama and French (1993) three-factor model ($R^2 = 93\%$, $RMSE = 0.786\%$).[28] According to these

[26] The estimate for δ is greater than 1 in 33% of the bootstrap replications.

[27] For the status quo, the adaptation, and the constant-parameter models (FORA(1), FORA(3), and FORA(4) models), the moment condition for the variance of the risk-free rate in equation (13) is only an approximation.

[28] The parameters for the Fama-French model are omitted because they do not have a structural interpretation.

results, the GDA discount factor can explain the cross-section of expected returns as accurately as the Fama-French model. This finding is important because structural models with consumption risk typically perform much worse than return-generated factors.

The performance of the FORA models is further examined in Table 6 which shows pricing errors for the five FORA specifications. According to these results, the alternative FORA models are rejected because they cannot price the size-B/M portfolios since the pricing errors are statistically significant. In contrast, the pricing errors for the disappointment model (FORA(5) model) are insignificant across all portfolios. These results are also confirmed by Figure 3, which shows fitted and sample expected returns for the FORA models. According to Figure 3, the GDA model is the only FORA model for which fitted and sample asset pricing moments are almost perfectly aligned.

6.3 Loss events in consumption growth

To explain the findings in Table 5, note that the explanatory power of reference-dependent preferences lies in the ability of these models to correctly characterize loss events in consumption growth, i.e., periods during which consumption growth is below the reference point. For the GDA model (FORA(5)), these periods are called disappointment events and happen whenever consumption growth is below its generalized certainty equivalent. According to the results in Table 5, disappointment events in consumption growth occur with 10% probability. In relation to disappointment events, loss events for the status quo reference point (FORA(1)) occur too rarely (6.2% probability), whereas loss events for the expectation, quick-adaptation, and constant-parameter models (FORA(2) - (4)) happen too often (50%, 51.2%, and 26.5% probability, respectively).

Figure 4 plots the dynamics of the various reference points for the alternative FORA models. The solid line in Figure 4 is consumption growth, and the dashed line shows expected consumption growth. According to Figure 4, when the reference level is equal to expected consumption growth (FORA(2) model), loss events should occur with a 50% probability because consumption growth is a normal random variable. Indeed, the probability of loss events for the FORA(2) model in Table 5 is exactly 50%. In this case, loss events happen so often that they overlap periods of economic growth and high stock market returns. These periods are not particularly important for asset prices, and the first-order risk aversion becomes irrelevant. The analysis is very similar when the reference point is either last period's consumption growth or a constant parameter (FORA(3) and FORA(4) models). In these models, loss events also happen too often.

In contrast, when the reference level is the status quo (FORA(1) model), there are too few loss events. Specifically, the reference level for the status quo model in Figure 4 is depicted by the horizontal line located at zero. In this case, consumption growth crosses the zero threshold with a 6.2% probability. The status quo model cannot explain asset prices because the zero threshold ignores a number of loss events in consumption growth that are important for asset prices (e.g., loss events in 1948, 1953, 1956, 1979, 1990, 1999).

Overall, the results in this section show that the assumptions regarding the location of the reference point determine the empirical fit of the FORA models. Specifically, I show that when the reference point is based on either the status quo assumption of Kahneman and Tversky (1979), the expectation model of Kőszegi and Rabin (2006), or a quickly adapting reference point as implied by Arkes et al. (2008), then the estimates for the price of downside consumption risk are insignificant. In this case, the reference-dependent model is equivalent to the traditional consumption-based framework with symmetric preferences, and cannot explain the cross-section of expected returns. Similar results hold when the reference point is a constant estimated by GMM, implying that reference points adapt over time.

In the next section, I examine the performance of disappointment models that use stock market returns as a proxy for returns on aggregate wealth instead of expressing the GDA discount factor in terms of consumption growth as in equation (6).

7. The market-based GDA discount factor

The results in the previous section show that a consumption-based model with disappointment aversion can explain the level and cross-sectional dispersion of expected stock returns as successfully as the Fama-French model. In this section, I show that expressing aggregate wealth returns in terms of consumption growth hugely improves the performance of disappointment models that use stock market returns to proxy for returns on aggregate wealth.

Using returns on aggregate wealth ($R_{W,t+1}$), the GDA stochastic discount factor from (3) can be rewritten as[29]

$$M_{t,t+1}^{GDA\ market} = \beta^{-\frac{\alpha}{\rho}} \left(\frac{C_{t+1}}{C_t}\right)^{-\alpha\frac{\rho-1}{\rho}} R_{W,t+1}^{-\frac{\alpha}{\rho}-1} \frac{1+\theta \mathbf{1}\{\beta^{\frac{1}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\rho-1}{\rho}} R_{W,t+1}^{\frac{1}{\rho}} \leq \delta\}}{\mathbb{E}_t\left[1+\theta\delta^{-\alpha}\mathbf{1}\left\{\beta^{\frac{1}{\rho}}\left(\frac{C_{t+1}}{C_t}\right)^{\frac{\rho-1}{\rho}} R_{W,t+1}^{\frac{1}{\rho}} \leq \delta\right\}\right]}. \quad (17)$$

In the above specification, $R_{W,t+1}$ denotes returns on aggregation wealth, i.e. returns on a claim which pays aggregate consumption as its dividends. However, such a claim is not traded in financial markets, and therefore returns on aggregate wealth are unobservable. To estimate this model, I follow the previous literature and proxy returns on total wealth with stock market returns since total wealth returns are hard to measure. One disadvantage of this approach is that I cannot use the variance of the risk-free rate to identify ρ because the risk-free rate is a function of consumption growth and wealth returns. Therefore, to estimate the market-based disappointment model, I set ρ equal to 1 as in Routledge and Zin (2010).[30]

7.1 GMM results for the market-based GDA discount factor

Table 7 shows estimation results for the market-based GDA model. The GMM moment conditions are the Euler equations for the risk-free asset, the stock market, and the six Fama-French portfolios. For the first market-based specification (market GDA(1)), θ is restricted to zero and investors have symmetric utility. For this model, the estimate of the risk aversion parameter is statistically significant and equal to 1.399. For the second market-based specification (market GDA(2)), α and θ are jointly estimated. In this case, the estimate of the disappointment aversion parameter is statistically significant and equal to 1.024, whereas the estimate of the risk aversion coefficient is

[29]Routledge and Zin (2010) and the proof of *Proposition 1* in Appendix A.

[30]Results for alternative values of the EIS are qualitatively similar to the ones shown here.

insignificant.

For these first two market-based GDA specifications, the GDA parameter δ is set equal to 1. I relax this restriction for the market-based GDA specifications (3), (4), and (5). However, for these specifications, I follow Ostrovnaya et al. (2006) and prespecify the value of the risk aversion parameter ($\alpha \in \{-1, 3, 15\}$) due to identification issues regarding the joint estimation of α, θ, and δ. The results for the market-based GDA specifications (3), (4), and (5) uncover a substitution effect between the risk and disappointment aversion parameters, similar to the one documented in Table 2. For example, as I increase the risk aversion parameter from -1 in specification (3) to 15 in specification (5), the disappointment aversion parameter decreases from 2.351 to -0.999. In untabulated results, I find that when the risk aversion parameter is greater than 1.5, the estimates for the disappointment aversion coefficient are negative.

According to the estimation results in Table 7, the GDA parameter δ is statistically significant across all market-based GDA models. However, similar to the consumption-based GDA models from Table 2, the results for the magnitude of the GDA in Table 7 are inconclusive. For example, in the market-based specification (3), the estimate of δ is greater than 1 ($\delta = 1.031$) while in specifications (4) and (5), the estimates of δ are equal to 0.916 and 0.940, respectively. These results are also confounded by the fact that, in the latter specifications, the estimates of the disappointment aversion parameter are negative.

Overall, the estimates for the market-based GDA models in Table 7 are quite consistent with the evidence for risky choices at the micro level. Specifically, the discount rate is less than 1 across all market-based models ($\beta = 0.947$ - 0.974), and the estimates of the disappointment aversion coefficient are low. In fact, the magnitude of the disappointment aversion parameter might be too low since the estimates for θ are negative in two out of the four market-based GDA specifications.[31] Next, I examine the fit of the market-based GDA model.

[31] For the market-based GDA specification (1), the disappointment aversion parameter is set equal to zero.

7.2 Pricing errors for the market-based GDA discount factor

According to the results in Table 7, all of the market-based GDA specifications are rejected by the J-test. Moreover, the R^2 for the market-based model without disappointment aversion (market GDA(1)) is negative (-1.000%) while the R^2 for the market-based GDA specification with disappointment aversion (market GDA(2)) is 12.663%. The performance of the market-based GDA model improves once I introduce the GDA parameter δ in the market specifications (3), (4), and (5). In this case, the R^2 ranges from 22.786% for specification (4) to 83.737% for specification (5). However, in both specifications, the estimates for the disappointment aversion parameter are negative.

Table 8 shows pricing errors for the five market-based GDA models. The pricing errors of the first four market-based models are economically and statistically significant. In contrast, the market-based GDA specification (5) performs better than the rest of the market models even though it cannot price the small-growth portfolio. Consistent with the results in Table 8, Figure 5 shows that the market-based GDA model cannot align fitted and sample expected returns unless the risk aversion parameter is high and the disappointment aversion coefficient is negative, as in the market-based specification (5).

Overall, the fit of the market-based GDA model is worse than the fit of the consumption-based GDA model from Table 2. In fact, the first four market-based GDA models in Table 7 perform worse than the consumption-based Epstein-Zin model of Table 2, which completely ignores disappointment aversion. This is because, at the annual frequency, the stock market portfolio does not account for important components of total wealth such as human capital (Lustig, Van Nieuwerburgh, and Verdelhan (2013)), whereas aggregate consumption is a better measure of economic activity.

Collectively, the results in this section run against the use of stock market returns as a proxy for total wealth returns at the annual frequency. In contrast, by solving the value function in terms of consumption growth as in equation (6), the fit of the consumption-based GDA model is superior to the fit of the market-based GDA model.

8. Aggregating first-order risk averse investors

The theoretical framework in this paper assumes identical preferences which can be aggregated due to the linear homogeneity of disappointment aversion. Nevertheless, Chapman and Polkovnichenko (2009) show that in models with first-order risk aversion, the equity premium and the risk-free rate are sensitive to preference heterogeneity. In addition, Easley and Yang (2012) use a dynamic model to show that the price impact of first-order risk averse investors is limited because the stock market participation of these investors shrinks at a very high speed.

Even though a heterogeneous agents model is outside the scope of this paper, Appendix D shows that disappointment aversion is equivalent to portfolio optimization with lower partial moments (conditional value-at-risk). Therefore, if investors use lower partial moments to optimize their portfolio holdings, then first-order risk aversion is a more realistic description of investor behavior than second-order risk aversion. Whether stock markets are populated by first-order or second-order risk averse investors is left for future research. Nevertheless, even if markets are populated by second-order risk averse investors, Epstein (1991) shows that suboptimal risk-sharing rules lead to aggregate preferences that exhibit first-order risk aversion.

9. Conclusion

This paper provides explicit solutions for the generalized disappointment aversion discount factor when consumption growth is a continuous random variable with constant volatility. These solutions allow me to run a wide range of comparative tests across alternative consumption-based asset pricing models and facilitate the identification of actual disappointment events in consumption growth.

To estimate the various consumption models, I employ an identification strategy that simultaneously fits consumption growth moments and Euler equations via a general GMM system. This strategy relies on the variance of the risk-free rate to identify the EIS when consumption growth is predictable and homoscedastic. Collectively, my results show that there is a substitution effect between disappointment and second-order risk aversion, and that the EIS estimates are less than 1 across all consumption-based models. However, my empirical setting cannot distinguish between Gul's (1991) original disappointment model, in which the reference point is equal to the certainty equivalent, and the generalized disappointment framework of Routledge and Zin (2010), in which the reference point is a multiple of the certainty equivalent.

In terms of model fit, comparative results suggest that a single-factor consumption-based model with disappointment aversion can explain the cross-section of expected returns as accurately as the Fama-French three-factor model. Moreover, the consumption-based disappointment model explains the cross-section of expected returns better than traditional consumption models or disappointment models in which stock market returns are used as a proxy for returns on total wealth.

Overall, this paper concludes that reference levels are not constant, and that the location of the reference payoff affects the empirical performance of consumption-based models with reference-dependent preferences. Specifically, this is the first paper to show that when the reference point is based on the status quo assumption of Kahneman and Tversky (1979), the expectation model of Kőszegi and Rabin (2006), or a quickly adapting reference point as implied by the evidence in Arkes et al. (2008), then downside consumption risk is not priced. In this case, the reference-dependent model is equivalent to the traditional consumption-based framework with symmetric preferences (no reference points), and cannot explain the cross-section of expected returns.

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Figures

Figure 1 Sample and fitted expected returns for the GDA discount factor

Figure 1 shows sample and fitted expected returns for the risk-free asset, the stock market, and the six size-B/M portfolios. Fitted expected returns are estimated according to the expression in (12) for the consumption-based GDA discount factor from equation (6) with different values for the risk aversion parameter α. The fitted value for the variance of the risk-free rate is given by the expression in (13). Figure 1 also shows sample and fitted expected returns for the CRRA and Epstein-Zin (EZ) models. Estimation results are shown in Table 2.

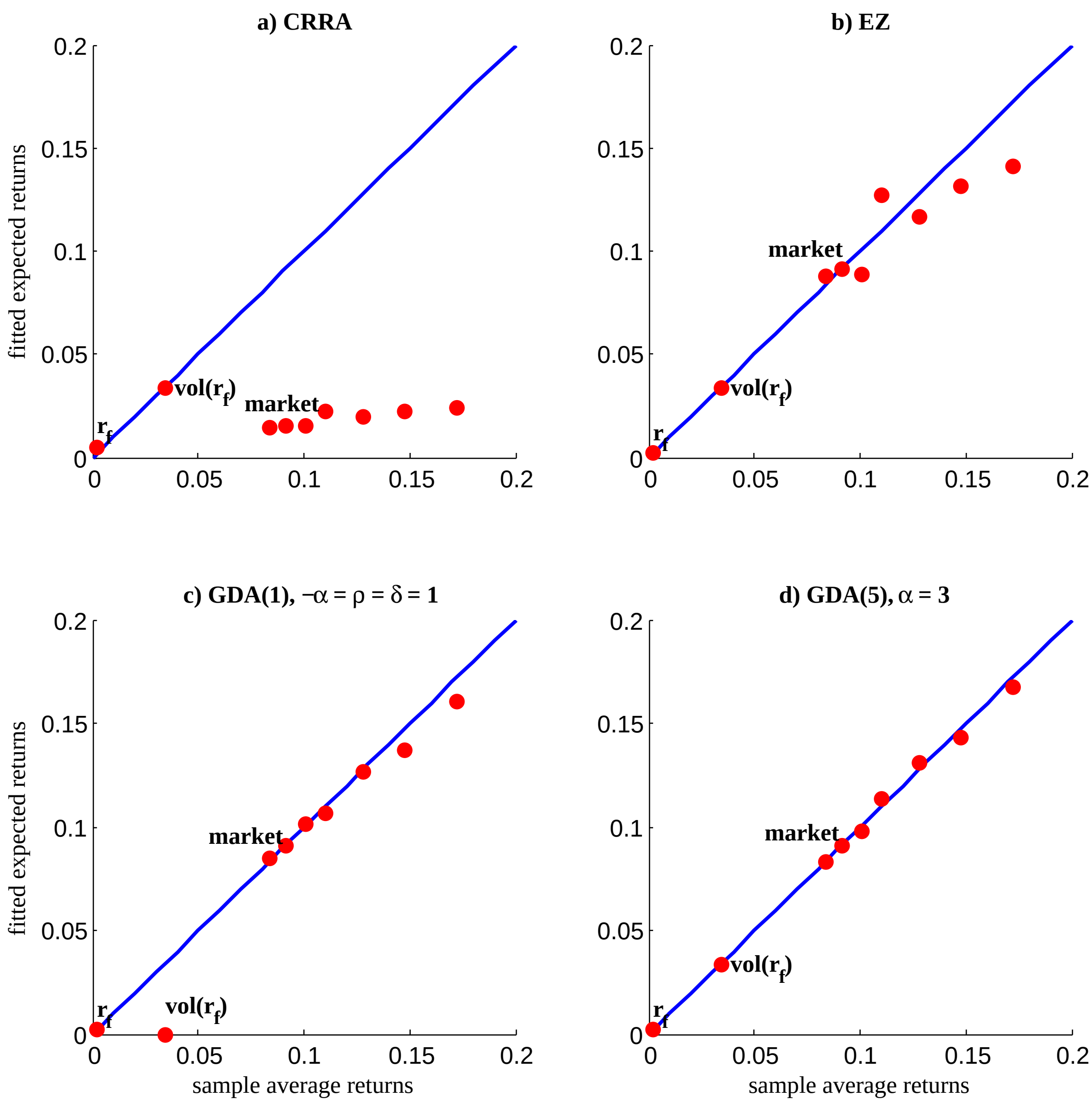


Figure 2 Consumption growth, disappointment events, and NBER recessions

Figure 2 shows disappointment events in consumption growth for the GDA discount factor from equation (6) when the risk aversion parameter is equal to 3 (GDA(5) model in Table 2). The solid line is consumption growth. The dashed line is the time-varying GDA certainty equivalent of consumption growth from the expression in (7). Disappointment events are highlighted by ellipses, and shaded areas are NBER recession dates. Estimation results for consumption growth moments and the disappointment threshold are shown in Table 2.

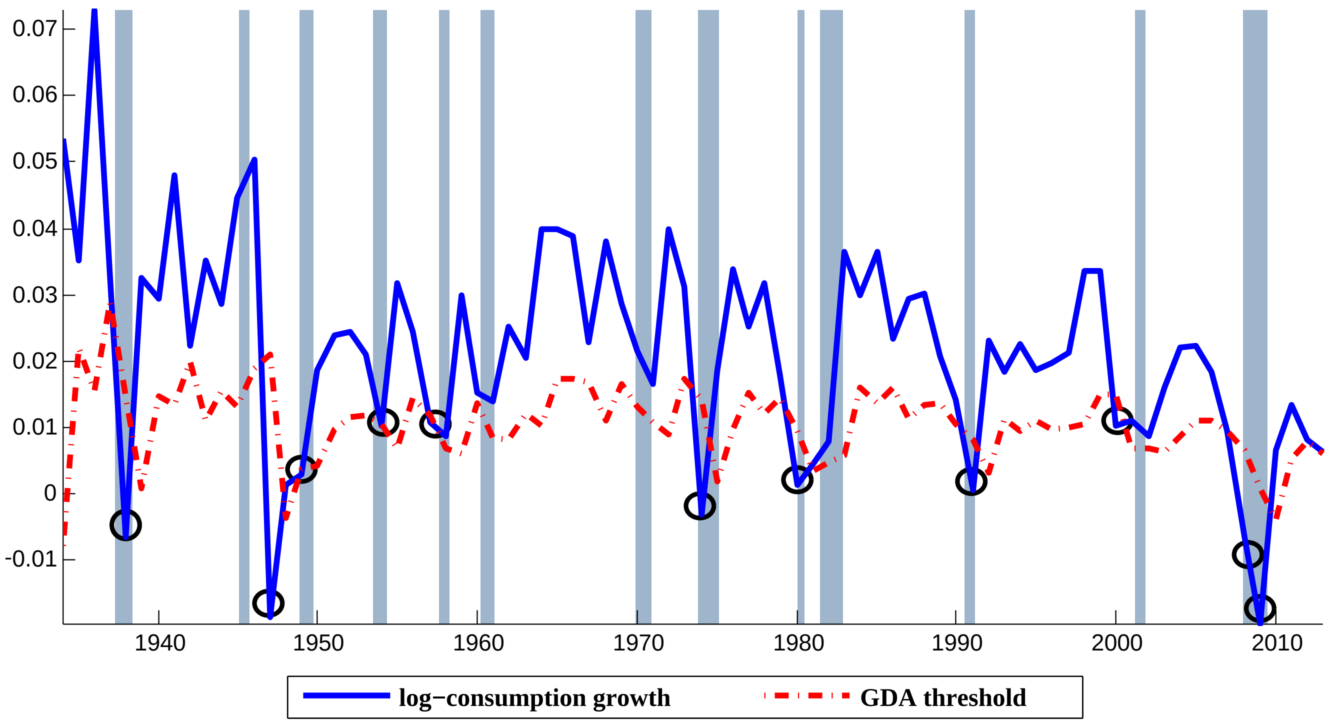


Figure 3 Sample and fitted expected returns for the FORA discount factor

Figure 3 shows sample and fitted expected returns for the risk-free asset, the stock market, and the six size-B/M portfolios. Fitted expected returns are estimated according to the expression in (12) for the FORA discount factor from equation (16) with alternative reference points for gains and losses. In Panel **a)**, the reference point is zero consumption growth ($d = 0$); in Panel **b)**, the reference point is expected consumption growth ($d = \mathbb{E}_t[\Delta c_{t+1}]$); in Panel **c)**, the reference point is a constant parameter estimated by GMM ($d = \hat{d}_0$); and in Panel **d)**, the reference point is the GDA certainty equivalent from equation (7). Estimation results are shown in Table 5.

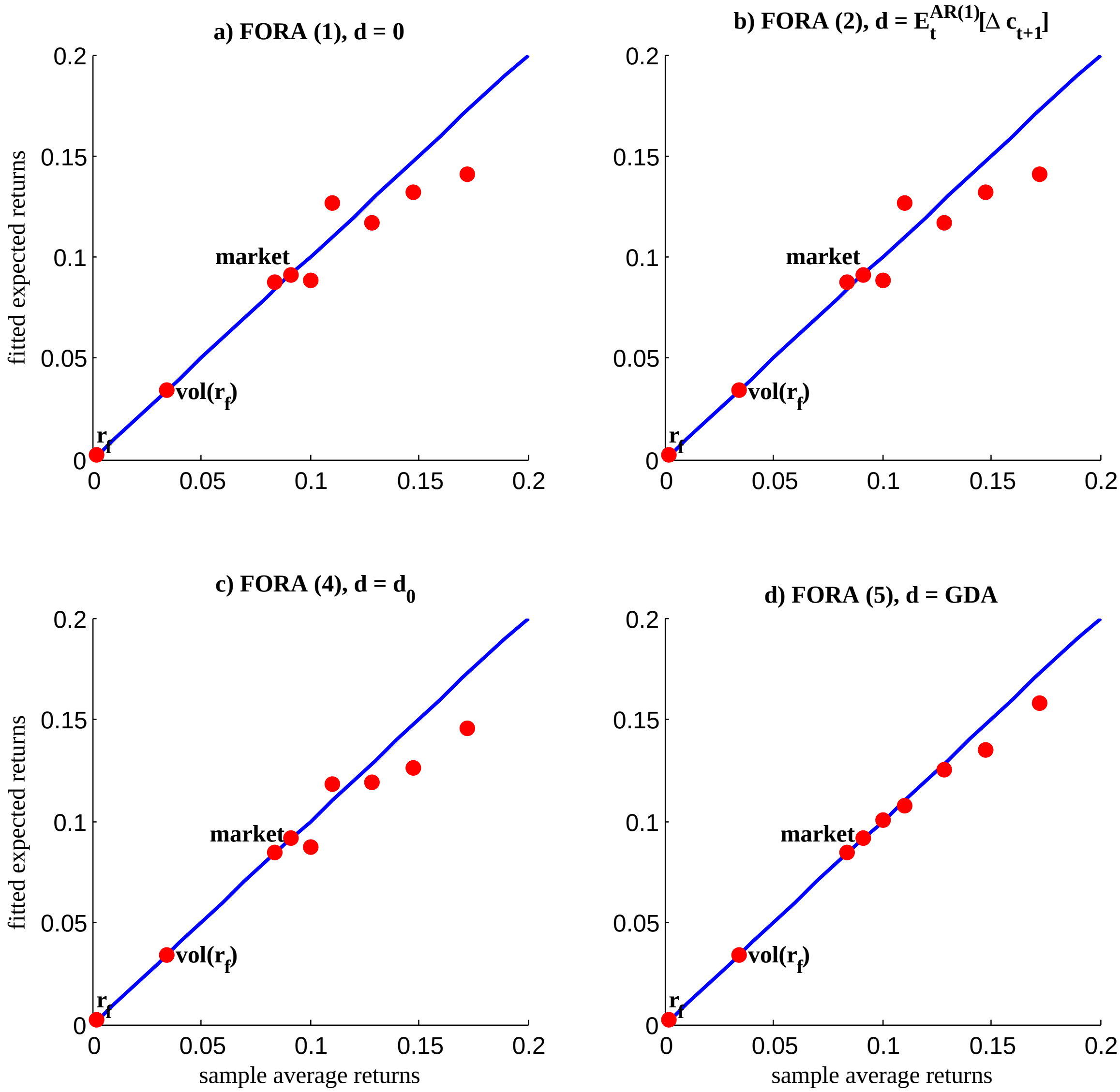


Figure 4 Consumption growth, loss events, and NBER recessions

Figure 4 shows loss events in consumption growth for the FORA discount factor from equation (16) with alternative reference points for gains and losses. The solid line is consumption growth. The dotted horizontal line shows the zero reference point according to the status quo model (FORA(1) model in Table 6). The time-varying dashed line is expected consumption growth and shows the reference point according to the expectation model (FORA(2) model in Table 6). Finally, the horizontal dashed-dotted line shows the constant reference point estimated by GMM (FORA(4) model in Table 6). Shaded areas are NBER recession dates.

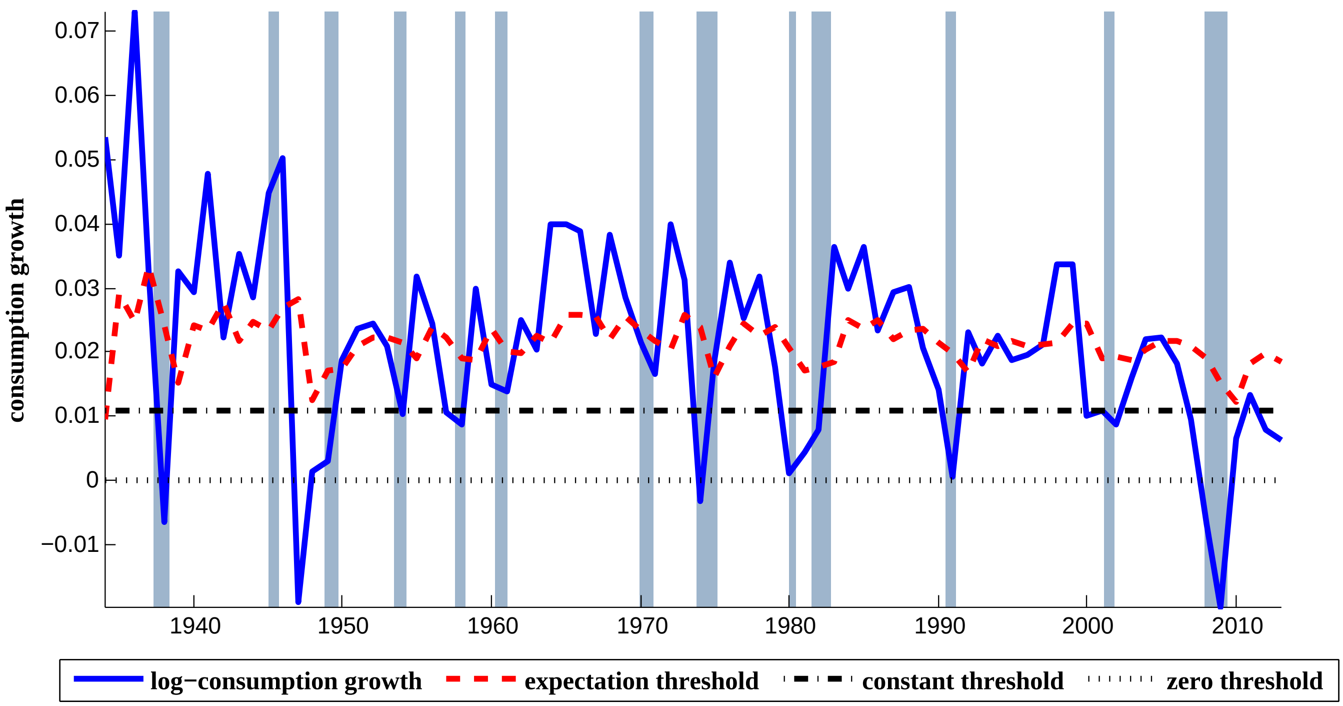
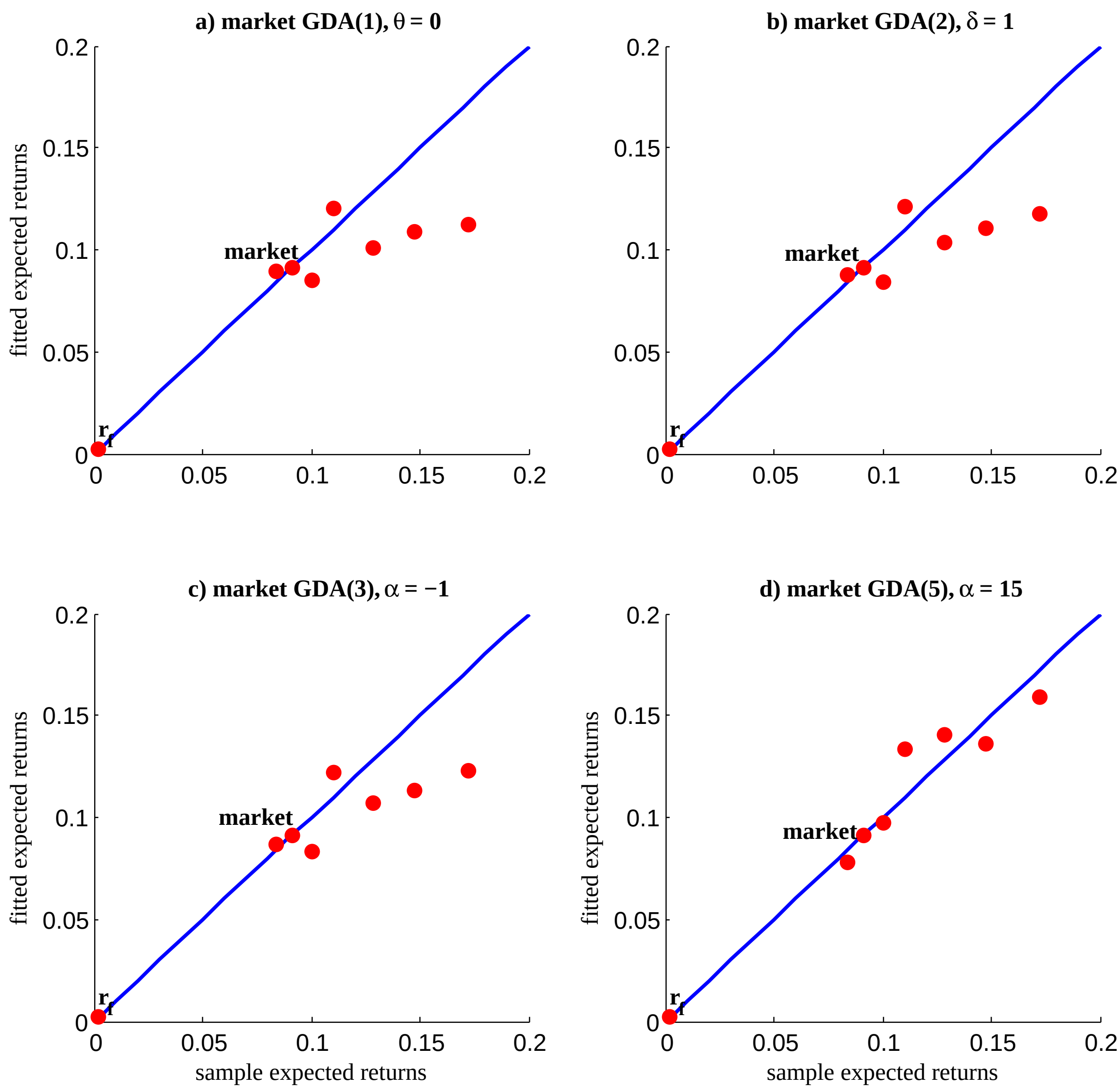


Figure 5 Sample and fitted returns for the market-based GDA discount factor

Figure 5 shows sample and fitted expected returns for the risk-free asset, the stock market, and the six size-B/M portfolios. Fitted expected returns are estimated according to the expression in (12) for the market-based GDA discount factor from equation (17) with different values for the risk aversion coefficient α. In the market-based GDA models, stock market returns are used as a proxy for aggregate wealth returns. Estimation results are shown in Table 7.



Tables

Table 1 Summary statistics for consumption growth and real asset returns

Table 1 shows summary statistics for the log-consumption growth (Δc_t), the log risk-free rate ($r_{f,t}$), and equity log-returns. Equity returns are for the stock market and the Fama-French six portfolios sorted on size and book-to-market. The sample period is 1933-2012.

				small			**big**		
	$\boldsymbol{\Delta c_t}$	$\boldsymbol{r_{f,t}}$	**market**	growth	medium B/M	value	growth	medium B/M	value
mean	2.16	0.17	7.17	6.81	11.03	12.78	6.44	8.07	9.74
volatility	1.55	3.41	18.16	27.45	23.88	25.37	18.35	17.81	21.91
autocorrelation	0.37	0.77	-0.05	-0.04	-0.04	-0.05	-0.02	-0.07	-0.20
Sharpe ratio			0.38	0.24	0.45	0.49	0.34	0.44	0.43

Table 2 GMM results for the consumption-based GDA discount factor

Table 2 shows estimation results for the CRRA, Epstein-Zin, and the GDA discount factors from equation (6) with different values for the risk aversion parameter α. The GMM moment conditions are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of the risk-free rate, and the unconditional Euler equations for the risk-free asset, the stock market, and the six size-B/M portfolios. Δc_t is consumption growth, μ_c ($\times 100$) is consumption growth mean, σ_c^2 ($\times 100$) is consumption growth variance, and $\phi_c \sigma^2$ ($\times 100$) is consumption growth autocovariance. β is the rate of time preference, α is the risk aversion parameter, ρ is equal to $1 - 1/\text{EIS}$, θ is the disappointment aversion coefficient, and δ is the GDA parameter. d_1 is the disappointment threshold from equation (7), and *disappointment probability* is the probability of disappointment events. Bootstrapped 95% confidence intervals are shown in brackets. *J-test*, *d.o.f.*, and *p-value* are the bootstrapped first-stage J-test (Hansen (1982)), the degrees of freedom, and the p-value that all moment conditions are jointly zero. $RMSE$ is the cross-sectional root mean square error ($\times 100$) for the market and the six size-B/M portfolios. R^2 is the cross-sectional R-square ($\times 100$) for the market and the six size-B/M portfolios.

			Generalized Disappointment Aversion (GDA)				
	CRRA $\alpha = -\rho$, $\theta = 0$	**Epstein-Zin** $\theta = 0$	**GDA(1)** $\alpha = -\rho = -\delta = -1$	**GDA(2)** $\alpha = -\delta = -1$	**GDA(3)** $\alpha = -1$	**GDA(4)** $\alpha = 1$	**GDA(5)** $\alpha = 3$
mean Δc_t ($\boldsymbol{\mu_c}$)	2.167 [1.91, 2.45]	2.169 [1.91, 2.45]	2.169 [1.91, 2.45]	2.169 [1.91, 2.45]	2.169 [1.91, 2.45]	2.169 [1.91, 2.45]	2.169 [1.91, 2.45]
variance Δc_t ($\boldsymbol{\sigma_c^2}$)	0.000 [0.000, 0.000]	0.023 [0.012, 0.024]	0.024 [0.010, 0.026]	0.023 [0.012, 0.031]	0.024 [0.012, 0.030]	0.023 [0.012, 0.032]	0.024 [0.013, 0.030]
autocov. Δc_t ($\boldsymbol{\phi_c \sigma_c^2}$)	0.000 [0.000, 0.005]	0.005 [0.001, 0.009]	0.006 [0.001, 0.010]	0.005 [0.002, 0.008]	0.006 [0.002, 0.009]	0.005 [0.001, 0.008]	0.008 [0.003, 0.012]
$\hat{\boldsymbol{\beta}}$	1.179 [0.97, 1.33]	1.159 [0.92, 1.34]	0.998 [0.98, 1.02]	1.069 [1.01, 1.16]	1.070 [1.02, 1.28]	1.092 [1.04, 1.32]	1.067 [1.04, 1.20]
$\hat{\boldsymbol{\alpha}}$		38.369 [27.94, 68.64]					
$\hat{\boldsymbol{\rho}}$	-7.218 [-10.96, 0.99]	-7.999 [-16.64, -2.54]		-8.224 [-18.36, -2.81]	-6.735 [-12.91, -2.85]	-8.419 [-15.08, -3.01]	-5.122 [-8.03, -2.01]
$\hat{\boldsymbol{\theta}}$			8.414 [5.84, 27.11]	9.879 [5.84, 26.91]	9.970 [6.43, 17.63]	8.563 [5.29, 17.57]	7.608 [5.84, 12.46]
$\hat{\boldsymbol{\delta}}$					0.997 [0.95, 1.00]	0.997 [0.96, 1.00]	1.000 [0.94, 1.00]
disappointment threshold ($\boldsymbol{d_1}$)			-0.868	-0.923	-0.840	-0.813	-0.789
disappointment probability			0.100	0.100	0.100	0.100	0.137
$\boldsymbol{J - test}$	1080.872	156.795	51.267	2.291	1.012	0.594	0.422
$\boldsymbol{d.o.f.}$	7	6	7	6	5	5	5
$\boldsymbol{p - value}$	0	0	0	0.891	0.961	0.988	0.994
$\boldsymbol{RMSE}$	10.296	1.593	0.626	0.659	0.667	0.688	0.281
$\boldsymbol{R^2}$	-1123.5	71.165	95.514	95.012	94.889	94.574	99.094

Table 3 GMM pricing errors for the consumption-based GDA discount factor

Table 3 shows mean pricing errors (×100) for the consumption-based GDA discount factor from equation (6) for different values of the risk-aversion parameter. Table 3 also shows mean pricing errors for the two nested consumption models (CRRA, Epstein-Zin). The test assets are the Fama-French six size-B/M portfolios. Table 3 does not report pricing errors for consumption growth moments, the risk-free rate, or stock market returns because these moments are perfectly fitted by all models (except for the CRRA and GDA(1) specifications). Bootstrapped t-statistics are in parenthesis. Estimation results are shown in Table 2.

	GMM pricing errors for the size-B/M portfolios					
	small-growth	**small-medium**	**small-value**	**big-growth**	**big-medium**	**big-value**
CRRA	8.817	12.406	14.768	6.883	8.491	10.771
$\alpha = -\rho$, $\theta = 0$	(6.29)	(19.80)	(16.26)	(13.87)	(12.01)	(8.45)
Epstein-Zin	-1.628	1.560	3.099	-0.415	1.241	1.170
$\theta = 0$	(-3.38)	(1.55)	(4.15)	(-0.97)	(2.33)	(2.95)
GDA(1)	0.364	1.081	1.185	-0.070	-0.059	0.166
$\alpha = -\rho = -\delta = -1$	(0.21)	(0.63)	(0.56)	(-0.04)	(-0.03)	(0.06)
GDA(2)	0.673	1.155	1.102	-0.000	-0.155	0.136
$\alpha = -\delta = -1$	(0.21)	(0.37)	(0.43)	(-0.005)	(-0.03)	(0.04)
GDA(3)	0.699	1.167	1.106	-0.000	-0.155	0.141
$\alpha = -1$	(0.68)	(0.45)	(0.22)	(-0.00)	(-0.15)	(0.12)
GDA(4)	0.510	1.188	1.257	-0.029	-0.076	0.229
$\alpha = 1$	(0.07)	(0.74)	(0.47)	(-0.02)	(-0.06)	(0.16)
GDA(5)	-0.262	0.393	0.435	0.042	0.255	-0.266
$\alpha = 3$	(-0.03)	(0.16)	(0.74)	(0.03)	(0.12)	(-0.31)

Table 4 Willingness-to-pay for the consumption-based GDA discount factor

Table 4 assesses the willingness-to-pay, i.e., the effective risk aversion, of a disappointment averse investor for different values of the risk and disappointment aversion parameters. The willingness-to-pay is defined as the difference between the mean and the certainty equivalent of a gamble with possible outcomes $\$100,000+\epsilon$ in which ϵ is a $N(0,\sigma_\epsilon)$ random variable. Table 4 shows willingness-to-pay calculations for different values of σ_ϵ. The preference parameters are from Table 2. Panel A shows willingness-to-pay calculations for time-separable preferences in which the risk aversion coefficient is simply the estimates of α in Table 2. Panel B shows willingness-to-pay calculations for non-separable preferences in which the effective risk aversion coefficient ($\tilde{\alpha}$) takes into account the persistence in consumption growth and the EIS. In this case, the effective risk aversion parameter is $\tilde{\alpha} = \left(\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c} - \rho\right)$.

Panel A: separable preferences

		Generalized Disappointment Aversion (GDA)				
σ_ϵ	**Epstein-Zin**	**GDA(1)**	**GDA(2)**	**GDA(3)**	**GDA(4)**	**GDA(5)**
	$\alpha = 38.363$	$\alpha = -\delta = -1$	$\alpha = -\delta = -1$	$\alpha = -1,\ \delta = 0.997$	$\alpha = 1,\ \delta = 0.997$	$\alpha = 3,\ \delta = 1.000$
	$\theta = 0$	$\theta = 8.414$	$\theta = 9.879$	$\theta = 9.970$	$\theta = 8.563$	$\theta = 7.608$
2,000	795	1,769	1,879	1,702	1,619	1,764
5,000	5,260	4,403	4,677	4,512	4,493	4,728
10,000	22,538	8,794	9,341	9,200	9,741	10,575
15,000	46,415	13,186	14,005	13,888	15,656	17,844
20,000	72,353	17,577	18,668	18,576	22,480	27,536

Panel B: non-separable preferences

		Generalized Disappointment Aversion (GDA)				
σ_ϵ	**Epstein-Zin**	**GDA(1)**	**GDA(2)**	**GDA(3)**	**GDA(4)**	**GDA(5)**
	$\tilde{\alpha} = 49.598$	$\tilde{\alpha} = -\delta = -1$	$\tilde{\alpha} = -2.792,\ \delta = 1$	$\tilde{\alpha} = -2.824,\ \delta = 0.997$	$\tilde{\alpha} = -0.309,\ \delta = 0.997$	$\tilde{\alpha} = 4.380,\ \delta = 1.000$
	$\theta = 0$	$\theta = 8.414$	$\theta = 9.879$	$\theta = 9.970$	$\theta = 8.563$	$\theta = 7.608$
2,000	1,045	1,769	1,812	1,631	1,592	1,773
5,000	6,648	4,403	4,305	4,128	4,213	4,777
10,000	25.813	8,794	8,838	7,663	8,515	10,779
15,000	49,274	13,186	10,562	10,380	12,723	18,363
20,000	74,031	17,577	12,483	12,283	16,821	28,783

Table 5 GMM results for the FORA discount factor

Table 5 shows estimation results for the FORA discount factor from equation (16) with alternative reference levels for gains and losses. The reference levels are: i) zero consumption growth, ii) expected consumption growth, iii) last period's consumption growth, iv) a constant parameter estimated by GMM, and v) the generalized disappointment aversion certainty equivalent from equation (7). The GMM moment conditions are the consumption growth mean, the consumption growth variance, the consumption growth autocovariance, the variance of risk-free rate, and the unconditional Euler equations for the risk-free asset, the stock market, and the six size-B/M portfolios. β is the rate of time preference, α is the risk aversion parameter, ρ is equal to $1 - 1/\text{EIS}$, θ is the first-order risk aversion aversion parameter, and d_0 is a constant reference point estimated by GMM. *loss probability* is the probability that consumption growth is below the reference point. Bootstrapped 95% confidence intervals are shown in brackets. *J-test*, *d.o.f.*, and *p-value* are the bootstrapped first-stage *J*-test (Hansen (1982)), the degrees of freedom, and the *p*-value that all moment conditions are jointly zero. *RMSE* is the cross-sectional root mean square error ($\times 100$) for the market and the six size-B/M portfolios. R^2 is the cross-sectional *R*-square ($\times 100$) for the market and the six size-B/M portfolios. *Fama-French* is the Fama and French (1993) three-factor model.

	First-Order Risk Aversion (FORA)					
reference level:	**FORA(1)** zero cons. growth	**FORA(2)** expected cons. growth	**FORA(3)** past cons. growth	**FORA(4)** constant parameter	**FORA(5)** GDA	**Fama-French**
mean Δc_t ($\boldsymbol{\mu_c \times 100}$)	2.169	2.169	2.169	2.169	2.169	
	[1.91, 2.45]	[1.91, 2.45]	[1.91, 2.45]	[1.91, 2.45]	[1.91, 2.45]	
variance Δc_t ($\boldsymbol{\sigma_c^2 \times 100}$)	0.023	0.023	0.023	0.023	0.023	
	[0.011, 0.024]	[0.011, 0.024]	[0.012, 0.024]	[0.012, 0.023]	[0.012, 0.028]	
autocov. Δc_t ($\boldsymbol{\phi_c \sigma_c^2 \times 100}$)	0.005	0.005	0.005	0.005	0.005	
	[0.002, 0.007]	[0.002, 0.007]	[0.002, 0.007]	[0.002, 0.008]	[0.002, 0.008]	
$\hat{\boldsymbol{\beta}}$	1.170	1.170	1.171	1.132	1.105	
	[0.91, 1.29]	[0.86, 1.31]	[0.84, 1.34]	[0.95, 1.34]	[1.05, 1.30]	
$\hat{\boldsymbol{\alpha}}$	39.202	39.087	39.011	5.044	3.325	
	[21.29, 73.51]	[17.25, 80.89]	[11.97, 79.75]	[1.23, 10.10]	[0.85, 7.23]	
$\hat{\boldsymbol{\theta}}$	-0.007	-0.003	0.002	8.488	7.232	
	[-0.018, 0.001]	[-0.007, 0.000]	[-0.0004, 0.007]	[3.25, 17.64]	[4.18, 15.37]	
$\hat{\boldsymbol{\rho}}$	-8.566	-8.564	-8.564	-8.564	-8.549	
	[-13.02, -2.02]	[-13.26, 5.65]	[-13.41, -0.55]	[-14.77, -3.21]	[-14.73 -3.27]	
$\hat{\boldsymbol{\delta}}$					0.996	
					[0.965, 1.005]	
constant reference point ($\boldsymbol{d_0 \times 100}$)				1.097		
				[1.06, 1.35]		
loss probability	0.062	0.500	0.512	0.265	0.100	
$\boldsymbol{J-test}$	52.580	60.648	64.783	21.413	0.632	
$\boldsymbol{d.o.f.}$	5	5	5	4	4	
$\boldsymbol{p-value}$	0.000	0.000	0.000	0.000	0.959	
$\boldsymbol{RMSE}$	1.592	1.594	1.594	1.436	0.734	0.786
$\boldsymbol{R^2}$	71.175	71.116	71.134	76.217	93.825	92.874

Table 6 GMM pricing errors for the FORA discount factor

Table 6 shows mean pricing errors ($\times 100$) for the FORA discount factor from equation (16) with alternative reference points for gains and losses. The test assets are the six Fama-French size-B/M portfolios. Table 6 does not report pricing errors for consumption growth moments, the risk-free rate, or stock market returns because these moments are perfectly fitted by all models due to the choice of the GMM weighting matrix. Bootstrapped t-statistics are in parenthesis. Estimation results are shown in Table 5.

	GMM pricing errors for the size-B/M portfolios					
	small-growth	**small-medium**	**small-value**	**big-growth**	**big-medium**	**big-value**
FORA(1)	-1.610	1.579	3.105	-0.400	1.225	1.168
zero cons. growth	(-7.117)	(2.37)	(7.59)	(-1.04)	(1.29)	(4.33)
FORA(2)	-1.610	1.581	3.108	-0.400	1.224	1.174
expected cons. growth	(-3.69)	(2.42)	(7.61)	(-1.23)	(1.26)	(2.45)
FORA(3)	-1.611	1.580	3.107	-0.399	1.223	1.173
past cons. growth	(-3.39)	(2.59)	(8.55)	(-0.96)	(2.57)	(2.76)
FORA(4)	-0.743	2.102	2.630	-0.107	1.324	0.886
constant parameter	(-1.18)	(2.11)	(6.82)	(-0.15)	(1.48)	(2.15)
FORA(5)	0.325	1.225	1.435	-0.054	0.014	0.332
GDA	(0.06)	(1.33)	(1.79)	(-0.04)	(0.01)	(0.34)

Table 7 GMM results for the market-based GDA discount factor

Table 7 shows estimation results for the market-based GDA discount factor from equation (17) in which stock market returns are used as a proxy for returns on aggregate wealth. The GMM moment conditions are the unconditional Euler equations for the risk-free asset, the stock market, and the six size-B/M portfolios. β is the rate of time preference, α is the risk aversion parameter, ρ is equal to $1 - 1/EIS$, θ is the disappointment aversion coefficient, and δ is the generalized GDA parameter. Bootstrapped 95% confidence intervals are shown in brackets. *J-test*, *d.o.f.*, and *p-value* are the bootstrapped first-stage J-test (Hansen (1982)), the degrees of freedom, and the p-value that all moment conditions are jointly zero. *RMSE* is the cross-sectional root mean square error ($\times 100$) and R^2 is the cross-sectional R-square ($\times 100$) for the market and the six size-B/M portfolios.

	Market-Based Generalized Disappointment Aversion				
	market GDA(1) $\theta = 0$, $\rho = 1$, $\delta = 1$	**market GDA(2)** $\rho = 1$, $\delta = 1$	**market GDA(3)** $\alpha = -1$, $\rho = 1$	**market GDA (4)** $\alpha = 3$, $\rho = 1$	**market GDA(5)** $\alpha = 15$, $\rho = 1$
$\hat{\boldsymbol{\beta}}$	0.953 [0.94, 0.96]	0.966 [0.83, 1.11]	0.974 [0.95, 0.98]	0.947 [0.93, 0.95]	0.962 [0.96, 0.97]
$\hat{\boldsymbol{\alpha}}$	1.399 [0.69, 1.77]	-0.0002 [-0.0001, -0.0002]			
$\hat{\boldsymbol{\theta}}$		1.024 [0.50, 1.27]	2.351 [2.02, 2.60]	-0.565 [-0.68, -0.44]	-0.999 [-0.999, -0.998]
$\hat{\boldsymbol{\delta}}$			1.031 [1.01, 1.09]	0.916 [0.85, 0.97]	0.940 [0.93, 0.94]
disappointment probability		0.387	0.400	0.225	0.225
$\boldsymbol{J - test}$	161.016	66.603	15.187	82.612	44.594
$\boldsymbol{d.o.f.}$	6	5	5	5	5
$\boldsymbol{p - value}$	0.000	0.000	0.009	0.000	0.000
$\boldsymbol{RMSE}$	2.970	2.761	2.531	2.597	1.191
$\boldsymbol{R^2}$	-1.000	12.663	26.580	22.786	83.737

Table 8 GMM pricing errors for the market-based GDA discount factor

Table 8 shows mean pricing errors (×100) for the market-based GDA discount factor from equation (17) in which stock market returns are used as a proxy for returns on aggregate wealth. The test assets are the six size-B/M portfolios. Table 8 does not report pricing errors for the risk-free rate or stock market returns because these moments are perfectly fitted by all models due to the choice of the GMM weighting matrix. Estimation results are shown in Table 7.

	GMM pricing errors for the size-B/M portfolios					
	small-growth	**small-medium**	**small-value**	**big-growth**	**big-medium**	**big-value**
market GDA(1)	-1.000	3.816	5.983	-0.537	1.543	2.782
$\theta = 0$, $\delta = 1$	(-1.89)	(3.74)	(6.12)	(-2.56)	(7.30)	(7.79)
market GDA(2)	-1.071	3.637	5.467	-0.396	1.668	2.483
$\delta = 1$	(-2.06)	(3.47)	(8.48)	(-1.64)	(6.91)	(4.21)
market GDA(3)	-1.130	3.397	4.927	-0.313	1.784	2.114
$\alpha = -1$	(-3.62)	(2.60)	(3.58)	(-1.41)	(6.17)	(3.83)
market GDA(4)	-1.128	3.367	5.325	-0.403	1.175	2.170
$\alpha = 3$	(-1.70)	(5.28)	(11.57)	(-2.15)	(5.81)	(5.17)
market GDA(5)	-2.267	1.091	1.289	0.584	0.355	-1.216
$\alpha = 15$	(-2.13)	(1.75)	(1.55)	(1.65)	(0.92)	(-1.84)

Appendix

Appendix A Proof of *Proposition 1*

To prove *Proposition 1*, I combine the linear structure of disappointment aversion with the AR(1) dynamics for consumption growth. The proof consists of four steps. First, I express the price-dividend ratio for a claim on aggregate consumption as a linear function of consumption growth. Second, I solve the disappointment aversion discount factor in terms of consumption growth. Then, I obtain the unconditional Euler equations for asset returns, and finally, I show that the fixed point problem for the disappointment reference point has a unique solution.

Price-dividend ratio of a claim on aggregate consumption

Due to the linear homogeneity of the objective function, equation (1) can be written as

$$J_t W_t = \max_{C_t,\ \{w_{i,t}\}_{i=1}^n} \left[(1-\beta)C_t^\rho + \beta\mu_t(J_{t+1}W_{t+1})^\rho\right]^{\frac{1}{\rho}},$$

where J_t is marginal utility. Using the budget constraint $W_{t+1} = (W_t - C_t)R_{W,t+1}$, the objective function becomes

$$J_t W_t = \max_{C_t,\ \{w_{i,t}\}_{i=1}^n} \left[(1-\beta)C_t^\rho + \beta(W_t - C_t)^\rho\mu_t(J_{t+1}R_{W,t+1})^\rho\right]^{\frac{1}{\rho}},$$

in which $R_{W,t+1}$ are wealth returns. The first order conditions for C_t imply that

$$(1-\beta)\rho C_t^{\rho-1} - \beta\rho(W_t - C_t)^{\rho-1}\mu_t(J_{t+1}R_{W,t+1})^\rho = 0.$$

Dividing by $W_t^{\rho-1}$, we obtain

$$(1-\beta)\Big(\frac{C_t}{W_t}\Big)^{\rho-1} - \beta\Big(1 - \frac{C_t}{W_t}\Big)^{\rho-1}\mu_t(J_{t+1}R_{W,t+1})^\rho = 0. \tag{18}$$

Along an optimal consumption path, the following holds

$$J_t^\rho W_t^\rho = (1-\beta)C_t^\rho + \beta(W_t - C_t)^\rho \mu_t(J_{t+1}R_{W,t+1})^\rho.$$

Dividing again by W_t^ρ, we get that

$$J_t^\rho = (1-\beta)\Big(\frac{C_t}{W_t}\Big)^\rho + \beta\Big(1-\frac{C_t}{W_t}\Big)^\rho \mu_t(J_{t+1}R_{W,t+1})^\rho. \tag{19}$$

Equations (18) and (19) imply that

$$J_t^\rho = (1-\beta)\Big(\frac{C_t}{W_t}\Big)^{\rho-1}. \tag{20}$$

We can substitute the above relation into equation (18) to get

$$(1-\beta)\Big(\frac{C_t}{W_t}\Big)^{\rho-1} - \beta(1-\beta)\Big(1-\frac{C_t}{W_t}\Big)^{\rho-1}\mu_t\Big[\Big(\frac{C_{t+1}}{W_{t+1}}\Big)^{(\rho-1)/\rho}R_{W,t+1}\Big]^\rho = 0.$$

Using the budget constraint once more, the first-order condition for optimal consumption becomes

$$(1-\beta)\Big(\frac{C_t}{W_t}\Big)^{\rho-1} - \beta(1-\beta)\Big(1-\frac{C_t}{W_t}\Big)^{\rho-1}\mu_t\Big[\Big(\frac{C_{t+1}}{(W_t-C_t)R_{W,t+1}}\Big)^{(\rho-1)/\rho}R_{W,t+1}\Big]^\rho = 0,$$

which simplifies into

$$\beta\mu_t\Big[\Big(\frac{C_{t+1}}{C_t}\Big)^{(\rho-1)/\rho}R_{W,t+1}^{1/\rho}\Big]^\rho = 1. \tag{21}$$

Let $P_{C,t} = W_t - C_t$ be the price for a claim on aggregate consumption. We can use the price-dividend identity

$$R_{W,t+1} = \frac{C_{t+1}}{C_t}\frac{P_{C,t+1}/C_{t+1}+1}{P_{C,t}/C_t}, \tag{22}$$

to recast equation (21) as

$$\frac{1}{\beta}^{\frac{1}{\rho}}\Big(\frac{P_{C,t}}{C_t}\Big)^{\frac{1}{\rho}} = \mu_t\Big[\frac{C_{t+1}}{C_t}\Big(\frac{P_{C,t+1}}{C_{t+1}}+1\Big)^{1/\rho}\Big]. \tag{23}$$

Following Campbell and Shiller (1988), a log-linear approximation for the price-dividend identity in equation (22) around the point $\overline{pc}$ is given by

$$\begin{aligned} logR_{W,t+1} &\approx \kappa_{c,0} + \kappa_{c,1}p_{c,t+1} - p_{c,t} + \Delta c_{t+1}, \text{ since} \\ log(P_{C,t+1}/C_{t+1}+1) &\approx \kappa_{c,0} + \kappa_{c,1}log(P_{C,t+1}/C_{t+1}) \end{aligned} \tag{24}$$

in which $p_{c,t} = log\frac{P_{C,t}}{C_t}$, $\kappa_{c,1} = \frac{e^{\overline{pc}}}{1+e^{\overline{pc}}} < 1$, and $\kappa_{c,0} = log(1+e^{\overline{pc}}) - \kappa_{c,1}\overline{pc}$.

We now conjecture that the log price-dividend ratio is linear in consumption growth $p_{c,t} = \mu_v + \phi_v \Delta c_t$. Using the definition of the certainty equivalent μ_t from equation (2), equation (23) becomes

$$\begin{aligned} \frac{\alpha}{\rho}(log\beta - pc_t) = log\mathbb{E}_t\Big[& e^{-\alpha\Delta c_{t+1} - \frac{\alpha}{\rho}(\kappa_{c,0}+\kappa_{c,1}pc_{t+1})} \times \\ & \frac{1+\theta\mathbf{1}\big\{\frac{C_{t+1}}{C_t}\big(\frac{P_{C,t+1}}{C_{t+1}}+1\big)^{\frac{1}{\rho}} \le \delta\mu_t\big[\frac{C_{t+1}}{C_t}\big(\frac{P_{C,t+1}}{C_{t+1}}+1\big)^{\frac{1}{\rho}}\big]\big\}}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}\mathbb{E}_t\Big[\mathbf{1}\big\{\frac{C_{t+1}}{C_t}\big(\frac{P_{C,t+1}}{C_{t+1}}+1\big)^{\frac{1}{\rho}} \le \delta\mu_t\big[\frac{C_{t+1}}{C_t}\big(\frac{P_{C,t+1}}{C_{t+1}}+1\big)^{\frac{1}{\rho}}\big]\big\}\Big]}\Big]. \end{aligned}$$

We can pin down μ_t from equation (23), and use the log-linearized price-dividend identity (24) to simplify the expression inside the disappointment indicator. Further, the partial moments property for a standard normal variable $\epsilon_{c,t+1}$ and real numbers $\big[\alpha,\, \rho,\, \kappa_{c,1},\, \phi_{v,1},\, \phi_c,\, \sigma_c,\, \tilde{d}_1\big]$ implies that[32]

$$\begin{aligned} &\mathbb{E}_t[e^{-\big(\frac{\alpha}{\rho}\kappa_{c,1}\phi_{v,1}+\alpha\big)\sqrt{1-\phi_c^2}\sigma_c\epsilon_{c,t+1}}\mathbf{1}\{\epsilon_{c,t+1} \le \tilde{d}_1\}] = \\ &e^{\frac{1}{2}[\big(\frac{\alpha}{\rho}\kappa_{c,1}\phi_{v,1}+\alpha\big)\sqrt{1-\phi_c^2}\sigma_c]^2} N\Big(\tilde{d}_1 + \alpha\big(\frac{\kappa_{c,1}\phi_{v,1}}{\rho}+1\big)\sqrt{1-\phi_c^2}\sigma_c\Big). \end{aligned}$$

Using the above result, the conjecture that $pc_{c,t} = \mu_v + \phi_v \Delta c_t$, and the AR(1) dynamics for

[32]Winkler, Roodman, and Britney (1972).

consumption growth, equation (23) becomes

$$\frac{\alpha}{\rho}log\beta - \frac{\alpha}{\rho}(\mu_v + \phi_v \Delta c_t) = -\alpha\big(\mu_c(1-\phi_c) + \phi_c \Delta c_t\big) - \frac{\alpha}{\rho}\kappa_{c,0} - \frac{\alpha}{\rho}\kappa_{c,1}\mu_v$$
$$-\frac{\alpha}{\rho}\kappa_{c,1}\phi_v\mu_c(1-\phi_c) - \frac{\alpha}{\rho}\kappa_{c,1}\phi_v\phi_c\Delta c_t + log\Big(1+\theta N\big(\tilde{d}_1 + \big(\frac{\alpha}{\rho}\kappa_{c,1}\phi_v + \alpha\big)\sqrt{1-\phi_c^2}\sigma_c\big)\Big)$$
$$-log\Big(1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\} + \theta\delta^{-\alpha}N(\tilde{d}_1)\Big) + \frac{1}{2}\big[\big(\frac{\alpha}{\rho}\kappa_{c,1}\phi_v + \alpha\big)\sqrt{1-\phi_c^2}\sigma_c\big]^2, \quad (25)$$

where $N()$ is the standard normal c.d.f., and $\tilde{d}_1$ is the threshold for disappointment defined as

$$\tilde{d}_1 = \frac{\rho log\delta - log\beta + \mu_v + \phi_v\Delta c_t - \kappa_{c,0} - \kappa_{c,1}\mu_v - (\kappa_{c,1}\phi_v + \rho)[\mu_c(1-\phi_c) + \phi_c\Delta c_t]}{(\kappa_{c,1}\phi_v + \rho)\sqrt{1-\phi_c^2}\sigma_c}. \quad (26)$$

We can now use the method of undetermined coefficients to find the values for μ_v and ϕ_v. First, we collect consumption growth terms ignoring the terms $log\Big(1+\theta N\big(\tilde{d}_1+\big(\frac{\alpha}{\rho}\kappa_{c,1}\phi_v+\alpha\big)\sqrt{1-\phi_c^2}\sigma_c\big)\Big)$ and $log\Big(1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}N(\tilde{d}_1)\Big)$ in equation (25). Then, we solve for ϕ_v to get

$$\phi_v = \frac{\rho\phi_c}{1-\kappa_{c,1}\phi_c}. \quad (27)$$

For the above value of ϕ_v, all Δc_t terms in equation (26) vanish, and $\tilde{d}_1$ becomes a function of constant terms alone. Also, for the above value of ϕ_v, the term $\kappa_{c,1}\phi_v/\rho + 1$ is positive since consumption growth is stationary ($\phi_c \in (-1,1)$) and $\kappa_{c,1} < 1$.

Collecting constant terms in (25), the solution for μ_v is given by

$$\mu_v = \frac{1}{1-\kappa_{c,1}}\Big[log\beta + \kappa_{c,0} + (\kappa_{c,1}\phi_v + \rho)\mu_c(1-\phi_c) - \frac{1}{2}\frac{\alpha}{\rho}\big[(\kappa_{c,1}\phi_v + \rho)\sqrt{1-\phi_c^2}\sigma_c\big]^2$$
$$-\frac{\rho}{\alpha}log\Big(1+\theta N\big(\tilde{d}_1 + \frac{\alpha}{\rho}(\kappa_{c,1}\phi_v + \rho)\sqrt{1-\phi_c^2}\sigma_c\big)\Big) + \frac{\rho}{\alpha}log\Big(1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\} + \theta\delta^{-\alpha}N(\tilde{d}_1)\Big)\Big],$$

and $\tilde{d}_1$ in equation (26) becomes the solution to the fixed point problem

$$\tilde{d}_1 = \frac{\rho log\delta}{(\kappa_{c,1}\phi_v + \rho)\sqrt{1-\phi_c^2}\sigma_c} - \frac{1}{2}\frac{\alpha}{\rho}(\kappa_{c,1}\phi_v + \rho)\sqrt{1-\phi_c^2}\sigma_c - \frac{log\left(\frac{1+\theta N\big(\tilde{d}_1 + \frac{\alpha}{\rho}(\kappa_{c,1}\phi_v+\rho)\sqrt{1-\phi_c^2}\sigma_c\big)}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}N(\tilde{d}_1)}\right)}{\frac{\alpha}{\rho}(\kappa_{c,1}\phi_v + \rho)\sqrt{1-\phi_c^2}\sigma_c}.$$

Using the solution for ϕ_v in (27), the fixed point problem for $\tilde{d}_1$ does not depend on ρ since

$$\tilde{d}_1 = \frac{(1-\phi_c\kappa_{c,1})log\delta}{\sqrt{1-\phi_c^2}\sigma_c} - \frac{1}{2}\frac{\alpha}{1-\kappa_{c,1}\phi_c}\sqrt{1-\phi_c^2}\sigma_c - \frac{log\Big(\frac{1+\theta N\left(\tilde{d}_1+\frac{\alpha}{1-\kappa_{c,1}\phi_c}\sqrt{1-\phi_c^2}\sigma_c\right)}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}N(\tilde{d}_1)}\Big)}{\frac{\alpha}{1-\kappa_{c,1}\phi_c}\sqrt{1-\phi_c^2}\sigma_c}, \quad (28)$$

and we can rewrite μ_v as

$$\mu_v = \frac{1}{1-\kappa_{1,c}}\big[log\beta + \kappa_{0,c} + (\kappa_{1,c}\phi_v + \rho)\mu_c(1-\phi_c) + \tilde{d}_1(\kappa_{1,c}\phi_v+\rho)\sqrt{1-\phi_c^2}\sigma_c - \rho log\delta\big].$$

Explicit solutions for the disappointment aversion stochastic discount factor

Turning back to the investor's optimization problem, the first-order conditions for portfolio weights are given by

$$\mathbb{E}_t\Big[J_{t+1}^{-\alpha}R_{W,t+1}^{-\alpha-1}\big(1+\theta\mathbf{1}\{J_{t+1}R_{W,t+1}\le\delta\mu_t(J_{t+1}R_{W,t+1})\}\big)(R_{i,t+1}-R_{n,t+1})\Big]=0.$$

Using the expression for J_{t+1} in equation (20) and the budget constraint, we obtain

$$\mathbb{E}_t\Big[C_{t+1}^{-\alpha\frac{\rho-1}{\rho}}R_{W,t+1}^{-\alpha/\rho-1}\big(1+\theta\mathbf{1}\{C_{t+1}^{\frac{\rho-1}{\rho}}R_{W,t+1}^{1/\rho}\le\delta\mu_t(C_{t+1}^{\frac{\rho-1}{\rho}}R_{W,t+1}^{1/\rho})\}\big)(R_{i,t+1}-R_{n,t+1})\Big]=0.$$

We can multiply both sides of the Euler equation by $\beta^{-\alpha/\rho}C_t^{\alpha\frac{\rho-1}{\rho}}$ and both sides of the inequality inside the disappointment indicator by $\beta^{1/\rho}C_t^{\frac{\rho-1}{\rho}}$ to get

$$\mathbb{E}_t\Big[\beta^{-\frac{\alpha}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{-\alpha\frac{\rho-1}{\rho}}R_{W,t+1}^{-\alpha/\rho-1}\big(1+\theta\mathbf{1}\{\beta^{\frac{1}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{1/\rho}\le\delta\mu_t\}\big)(R_{i,t+1}-R_{n,t+1})\Big]=0.$$

The choice of the n^{th} asset as the reference asset is arbitrary. The above equation could also be written as

$$\mathbb{E}_t\Big[\beta^{-\frac{\alpha}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{-\alpha\frac{\rho-1}{\rho}}R_{W,t+1}^{-\alpha/\rho-1}\big(1+\theta\mathbf{1}\{\beta^{\frac{1}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{1/\rho}\le\delta\mu_t\}\big)(R_{i,t+1}-R_{j,t+1})\Big]=0.$$

Multiplying both sides of the above equation with w_j and summing over j, we obtain

$$\mathbb{E}_t\Big[\beta^{-\frac{\alpha}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{-\alpha\frac{\rho-1}{\rho}}R_{W,t+1}^{-\alpha/\rho-1}\big(1+\theta\mathbf{1}\{\beta^{\frac{1}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{1/\rho}\leq\delta\mu_t\}\big)\big(\sum_{j\neq i}w_jR_{i,t+1}-\sum_{j\neq i}w_jR_{j,t+1}\big)\Big]=0.$$

Using the weights constraint $\Big(\sum_{j\neq i}w_j=1-w_i\Big)$, we get

$$\mathbb{E}_t\Big[\beta^{-\frac{\alpha}{\rho}}\big(\tfrac{C_{t+1}}{C_t}\big)^{-\alpha\frac{\rho-1}{\rho}}R_{W,t+1}^{-\alpha/\rho-1}\big(1+\theta\mathbf{1}\{\beta^{\frac{1}{\rho}}\big(\tfrac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{1/\rho}\leq\delta\mu_t\}\big)R_{i,t+1}\Big]=$$
$$\mathbb{E}_t\Big[\beta^{-\frac{\alpha}{\rho}}\big(\tfrac{C_{t+1}}{C_t}\big)^{-\alpha\frac{\rho-1}{\rho}}R_{W,t+1}^{-\alpha/\rho-1}\big(1+\theta\mathbf{1}\{\beta^{\frac{1}{\rho}}\big(\tfrac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{1/\rho}\leq\delta\mu_t\}\big)\textstyle\sum_j w_jR_{j,t+1}\Big].$$

The definition of the disappointment aversion certainty equivalent in equation (2) implies that we can write the right-hand side above as

$$\mathbb{E}_t\Big[\beta^{-\frac{\alpha}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{-\alpha\frac{\rho-1}{\rho}}R_{W,t+1}^{-\alpha/\rho-1}\big(1+\theta\mathbf{1}\{\beta^{\frac{1}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{1/\rho}\leq\delta\mu_t\}\big)R_{i,t+1}\Big]= \tag{29}$$
$$\mu_t\Big[\beta^{\frac{1}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{\frac{1}{\rho}}\Big]^{-\alpha}\mathbb{E}_t\Big[1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}\mathbf{1}\Big\{\beta^{\frac{1}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{\frac{1}{\rho}}\leq\delta\mu_t\Big\}\Big].$$

From equation (21), we know that $\mu_t\Big[\beta^{\frac{1}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{\frac{1}{\rho}}\Big]=1$, and therefore we can rewrite equation (29) as

$$\mathbb{E}_t\Big[\beta^{-\frac{\alpha}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{-\alpha\frac{\rho-1}{\rho}}R_{W,t+1}^{-\alpha/\rho-1}\frac{1+\theta\mathbf{1}\{\beta^{\frac{1}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{1/\rho}\leq\delta\}}{\mathbb{E}_t\Big[1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}\mathbf{1}\Big\{\beta^{\frac{1}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{\frac{1}{\rho}}\leq\delta\Big\}\Big]}R_{i,t+1}\Big]=1,$$

which implies that the disappointment aversion stochastic discount factor can be written as

$$M_{t,t+1}=\beta^{-\frac{\alpha}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{-\alpha\frac{\rho-1}{\rho}}R_{W,t+1}^{-\alpha/\rho-1}\frac{1+\theta\mathbf{1}\{\beta^{\frac{1}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{1/\rho}\leq\delta\}}{\mathbb{E}_t\Big[1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}\mathbf{1}\Big\{\beta^{\frac{1}{\rho}}\big(\frac{C_{t+1}}{C_t}\big)^{\frac{\rho-1}{\rho}}R_{W,t+1}^{\frac{1}{\rho}}\leq\delta\Big\}\Big]}.$$

Using the log-linearized price-dividend identity for returns on total wealth in equation (24), the stochastic discount factor can be further expressed as

$$M_{t,t+1} = e^{-\frac{\alpha}{\rho}log\beta - \frac{\alpha}{\rho}(\rho-1)\Delta c_{t+1} - (\frac{\alpha}{\rho}+1)[\kappa_{c,0}+\kappa_{c,1}(\mu_v+\phi_v\Delta c_{t+1})-(\mu_v+\phi_v\Delta c_t)+\Delta c_{t+1}]}$$
$$\times \frac{1+\theta\mathbf{1}\{\frac{1}{\rho}[log\beta+\kappa_{c,0}+\kappa_{c,1}\mu_v+(\kappa_{c,1}\phi_v+\rho)\Delta c_{t+1}-(\mu_v+\phi_v\Delta c_t)]\le log\delta\}}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{\frac{1}{\rho}[log\beta+\kappa_{c,0}+\kappa_{c,1}\mu_v+(\phi_v\kappa_{c,1}+\rho)\Delta c_{t+1})-(\mu_v+\phi_v\Delta c_t)]\le log\delta\}]}.$$

Finally, using the solutions for ϕ_v and μ_v, we conclude that

$$\begin{aligned} M_{t,t+1} &= e^{log\beta+(\rho-1)\Delta c_{t+1}+\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}\mu_c(1-\phi_c)+\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}d_1\sqrt{1-\phi_c^2}\sigma_c-[\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}]\Delta c_{t+1}+[\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}]\phi_c\Delta c_t} \\ &\times \frac{1+\theta\mathbf{1}\{\Delta c_{t+1}\le(1-\kappa_{c,1}\phi_c)log\delta+\mu_c(1-\phi_c)+\phi_c\Delta c_t+d_1\sqrt{1-\phi_c^2}\sigma_c\}}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{\Delta c_{t+1}\le(1-\kappa_{c,1}\phi_c)log\delta+\mu_c(1-\phi_c)+\phi_c\Delta c_t+d_1\sqrt{1-\phi_c^2}\sigma_c\}]}, \end{aligned}$$

where d_1 is given by

$$d_1 = \tilde{d}_1 - \frac{(1-\kappa_{c,1}\phi_c)log\delta}{\sqrt{1-\phi_c^2}\sigma_c}.$$

Unconditional Euler equations

We can write the Euler equation for the risk-free rate as

$$\mathbb{E}_t\Big[M_{t,t+1}R_{f,t+1}\Big]=1, \text{ or } \mathbb{E}_t\Big[\tilde{M}_{t,t+1}R_{f,t+1}\Big]=$$
$$1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}\mathbb{E}_t[\mathbf{1}\{\Delta c_{t+1}\le(1-\kappa_{c,1}\phi_c)log\delta+\mu_c(1-\phi_c)+\phi_c\Delta c_t+d_1\sqrt{1-\phi_c^2}\sigma_c\}],$$

in which

$$\begin{aligned} \tilde{M}_{t,t+1} &= e^{log\beta+(\rho-1)\Delta c_{t+1}+\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}\mu_c(1-\phi_c)+\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}d_1\sqrt{1-\phi_c^2}\sigma_c-[\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}]\Delta c_{t+1}+[\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}]\Delta c_t} \\ &\times \Big(1+\theta\mathbf{1}\{\Delta c_{t+1}\le(1-\kappa_{c,1}\phi_c)log\delta+(1-\phi_c)\mu_c-\phi_c\Delta c_t+d_1\sqrt{(1-\phi_c^2)\sigma_c^2}\}\Big). \end{aligned}$$

Taking unconditional expectations in both sides of the Euler equation and rearranging, we get that

$$\mathbb{E}\Big[M_{t,t+1}R_{f,t+1}\Big]=1, \tag{30}$$

where

$$
\begin{aligned}
M_{t,t+1} &= e^{log\beta+(\rho-1)\Delta c_{t+1}+\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}\mu_c(1-\phi_c)+\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}d_1\sqrt{1-\phi_c^2}\sigma_c-[\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}]\Delta c_{t+1}+[\frac{\alpha+\rho}{1-\kappa_{c,1}\phi_c}]\phi_c\Delta c_t} \\
&\times \frac{1+\theta\mathbf{1}\{\Delta c_{t+1}\leq(1-\kappa_{c,1}\phi_c)log\delta+\mu_c(1-\phi_c)+\phi_c\Delta c_t+d_1\sqrt{1-\phi_c^2}\sigma_c\}}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\mathbb{E}[\Delta c_{t+1}\leq(1-\kappa_{c,1}\phi_c)log\delta+\mu_c(1-\phi_c)+\phi_c\Delta c_t+d_1\sqrt{1-\phi_c^2}\sigma_c\}]}.
\end{aligned}
$$

Similar results hold for asset excess returns $R_{i,t+1}-R_{f,t+1}$

$$\mathbb{E}\Big[M_{t,t+1}(R_{i,t+1}-R_{f,t+1})\Big]=0.$$

Existence and uniqueness of the solution to the fixed-point problem for d_1

In order to show that the fixed point problem in (28) has a unique solution, consider the function $h:\mathbb{R}\to\mathbb{R}$ such that

$$h(x)=x+0.5A-B+\frac{1}{A}log\Big[\frac{1+\theta N\big(x+A\big)}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}N\big(x\big)}\Big].$$

with $A=\frac{\alpha}{1-\kappa_{c,1}\phi_c}\sqrt{1-\phi_c^2}\sigma_c$ and $B=\frac{(1-\kappa_{c,1}\phi_c)log\delta}{\sqrt{1-\phi_c^2}\sigma_c}$. For the function $h(x)$, we have that

$$\lim_{x\uparrow+\infty}h(x)=+\infty>0\quad\text{and}\quad\lim_{x\downarrow-\infty}h(x)=-\infty<0.$$

Therefore, because the function $h(x)$ is continuous in $\mathbb{R}$, there exists at least one solution to the equation $h(x)=0$ in $(-\infty,+\infty)$. Finally, even though it is hard to show uniqueness of solution algebraically, Figure A shows that $h(x)$ is strictly increasing for different values of θ while the rest of the parameters are kept constant and equal to the estimates from Table 2. Similar results hold if θ is fixed and I vary the rest of the parameters.

Figure A. Uniqueness of the solution for d_1

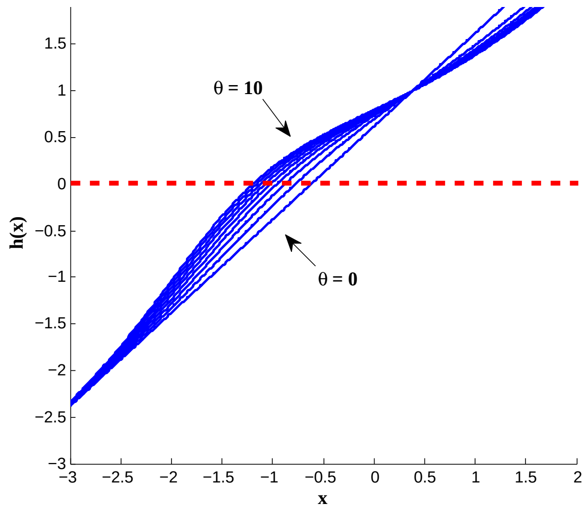


Figure A shows that the function $h(x) = x + 0.5A - B + \frac{1}{A} log\Big[\frac{1+\theta N\big(x+A\big)}{1-\theta(\delta^{-\alpha}-1)\mathbf{1}\{\delta>1\}+\theta\delta^{-\alpha}N\big(x\big)}\Big]$ is monotonically increasing in x as I vary θ and hold A and B fixed.

Appendix B Consistency of non-differentiable GMM estimators

In order to prove consistency and asymptotic normality of GMM estimators, standard applications require differentiability of the GMM objective function. However, continuity and differentiability break down when the GMM moment conditions include indicator functions. In this section, I use the results in Andrews (1994) and Newey and McFadden (1994) to show consistency and asymptotic normality for the GDA estimates, even if the GDA discount factor is not continuous.

Let z_t be a vector of random variables, x a vector of parameters, $q(z_t, x)$ a vector-valued function and W a positive-definite, symmetric matrix. The GMM objective function is given by

$$Q_0 = \mathbb{E}\big[q(z_t, x)\big]' W \ \mathbb{E}\big[q(z_t, x)\big], \tag{31}$$

and its sample analog

$$\hat{Q}_T = \Big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\Big]'\hat{W}\Big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\Big]. \tag{32}$$

Let x_0 be the minimizer in (31), and $\hat{x}_T$ the minimizer in (32). For the disappointment aversion model, $z_t = \big[\Delta c_{t+1}, \Delta c_t, \{R_{i,t+1}\}_{i=1}^{n-1}, R_{f,t+1}\big]$, $x = [\mu_c, \phi_c\sigma_c^2, \sigma_c^2, \beta, \rho, \theta, \alpha, \delta]$, and

$$q(z_t, x) = \begin{bmatrix} \Delta c_{t+1} - \mu_c \\ \Delta c_{t+1}^2 - \mu_c^2 - \sigma_c^2 \\ \Delta c_{t+1}\Delta c_t - \mu_c^2 - \phi_c\sigma_c^2 \\ (logR_{f,t+1})^2 - \mathbb{E}[logR_{f,t+1}]^2 - (1-\rho)^2\phi_c^2\sigma_c^2 \\ M_{t,t+1}R_{f,t+1} - 1 \\ M_{t,t+1}\big(R_{i,t+1} - R_{f,t+1}\big) \text{ for } i = 1, 2, ..., n-1 \end{bmatrix}.$$

Economic theory suggests that for disappointment averse investors $\beta \in (0, 1)$, while θ, α, ρ, and δ cannot assume infinite values, and are bounded away from infinity by real numbers B_θ, B_α and B_ρ. I also assume that Δc_t is stationary and therefore μ_c is a real number, ϕ_c takes values in $(-1, 1)$, and σ_c^2 is a positive number. For $\kappa_{c,1} \in (0, 1)$ and Δc_{t+1} stationary, it also follows that $1 - \phi_c\kappa_{c,1} > 0$. All these restrictions imply that the vector of parameters takes values in a compact space $\mathbf{X} \in \mathbb{R}^7$. Finally, I assume that z_t is characterized by a continuous probability distribution function and a well-defined moment generating function $\forall\, x \in \mathbf{X}$.

Identification: I assume that the GMM objective function in (31) satisfies the conditions of Lemma 2.3 in Newey and McFadden (1994), so that x_0 is globally identified:

1. $\mathbb{E}[q(z_t, x_0)] = 0$
2. $W\mathbb{E}[q(z_t, x_0)] \neq 0$ for $x \neq x_0$.

For the emprical part of this paper, I use the variance of the risk-free rate to identify ρ. Moreover, for the first set of empirical tests, I restrict the value of the risk aversion coefficient α in order to address any concerns that θ and α cannot be jointly identified.[33] Finally, I consider a grid of initial values for the minimization of the GMM objective function to ensure that the simplex algorithm does not capture local minima.

Consistency: For consistency of GMM estimators when the GMM objective function is not continuous, the reader is referred to Theorem 2.6 in Newey and McFadden (1994). This theorem requires that:

1. z_t is stationary and ergodic
2. $\hat{W} \xrightarrow{p} W$, W is positive definite, and $W\mathbb{E}[g(z, x_0)] = 0$ only if $x = x_0$
3. $\mathbf{X}$ is compact
4. $q(z_t, x)$ is continuous with probability one.
5. $\mathbb{E}\left[sup_{x\in\mathbf{X}}||q(z_t, x)||\right] < +\infty$

If all the above conditions are met, then $\hat{x}_T \xrightarrow{p} x_0$. Stationarity and ergodicity are reasonable properties for the random variable $z_t = \left[\Delta c_{t+1}, \Delta c_t, \{R_{i,t+1}\}_{i=1}^{n-1}, R_{f,t+1}\right]$ at the annual frequency. The second condition is satisfied because the first-stage GMM weighting matrix is constant, and equal to a diagonal matrix with prespecified elements. Moreover, according to the identification assumption above, the GMM objective function has a unique minimizer x_0 which can be identified. The third condition is satisfied because economic theory and stationarity of Δc_{t+1} suggest that the parameter space $\mathbf{X}$ is compact. The fourth condition is also satisfied because the only point of discontinuity in (33) is

$$\Delta c_{t+1} = (1 - \kappa_{c,1}\phi_c)\delta + \mu_c(1 - \phi_c) + \phi_c \Delta c_t + d_1\sqrt{1 - \phi_c^2}\sigma_c.$$

However, this discontinuity is a zero-probability event $\forall x \in \mathbf{X}$ because consumption growth is a continuous random variable. Finally, condition 5 above is satisfied because $\mathbf{X}$ is compact, and the distribution of z_t has a well-defined moment generating function $\forall x \in \mathbf{X}$.

[33] See also the discussion in Newey and McFadden (1994, p. 2127) on the Hansen and Singleton (1982) model.

Asymptotic normality: Theorems 7.2 and 7.3 in Newey and McFadden (1994) provide conditions for asymptotic normality of GMM estimates when the GMM objective function is not continuous. These conditions are

1. $\left[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\right]'\hat{W}\left[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\right] \leq \inf_{x\in\mathbf{X}} \left[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\right]'\hat{W}\left[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\right]$

2. $\hat{W} \xrightarrow{p} W$, W is positive definite

3. $\hat{x} \xrightarrow{p} x_0$

4. x_0 is in the interior of $\mathbf{X}$

5. $\mathbb{E}[g(z, x_0)] = 0$

6. $\sqrt{T}\left[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x_0)\right] \xrightarrow{d} N(0, \Sigma)$

7. $\mathbb{E}[g(z, x)]$ is differentiable at x_0 with derivative G, and $G'WG$ is non-singular

8. for $\delta_N \to 0$, then

$$\sup_{||x-x_0||\leq\delta_n} \frac{\sqrt{n}\left|\left|\left[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)\right] - \left[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x_0)\right] - \mathbb{E}\left[g(z, x_0)\right]\right|\right|}{1 + \sqrt{n}||x - x_0||} \xrightarrow{p} 0$$

If all the above conditions are met, then

$$\sqrt{T}(\hat{x}_T - x_0) \xrightarrow{d} N\big(0, (G'WG)^{-1}G'WSWG(G'WG)^{-1}\big).$$

The first condition is related to identification. The second condition is satisfied since $\hat{W}$ is a diagonal matrix with prespecified elements. The third condition is satisfied by the consistency theorem above. Conditions 4, 5, and 6 are standard GMM assumptions. The seventh condition is satisfied provided that the joint probability density function for asset returns and consumption growth is continuous, and the moment generating function is well-defined. The critical condition for asymptotic normality is condition 8, the stochastic equicontinuity condition.

Theorem 1 of Andrews (1994) provides primitive conditions for stochastic equicontinuity. These conditions are related to Pollard's entropy condition (Pollard (1984)). Fortunately, the GMM function for the disappointment model in equation (33) is a mixture of functions that satisfy this entropy condition. According to Theorem 2 in Andrews, indicator functions, which are "type I" functions, satisfy Pollard's conditions. A second class of functions which also satisfy Pollard's conditions are functions that depend on a finite number of parameters and are Lipschitz-continuous with respect to these parameters. These functions are called "type II" functions.[34]

The GMM vector-valued function $q(z_t, x)$ in equation (33) consists of linear and exponential terms, which, in turn, are functions of a finite number of preference parameters. Exponential functions are only locally Lipschitz-continuous. However, the GMM function in (33) is Lipschitz-continuous because the parameter space $\mathbf{X}$ is compact, and, therefore, the rate of change for all exponential functions in (33) remains bounded. Overall, according to Theorem 2 in Andrews (1994), the disappointment aversion GMM function in (33) includes terms that individually satisfy Pollard's entropy condition.

According to Theorem 3 in Andrews (1994), elementary operations between "type I" and "type II" functions result in functions that also satisfy Pollard's entropy condition. Hence, the disappointment aversion GMM function in (33), which is a product of "type I" and "type II" functions, satisfies the stochastic equicontinuity condition. Therefore, the GMM estimates for the disappointment model are asymptotically normally distributed. This discussion confirms that even though $q(z_t, x)$ in (33) is not continuous with respect to x, standard asymptotic results can still be applied provided that certain regularity conditions are satisfied. These conditions are in general associated with "continuity" and "differentiability" of the function $\mathbb{E}\big[q(z_t, x)\big]$ rather than the function $q(z_t, x)$ itself.

Finally, even if $q(z_t, x)$ is not continuously differentiable, we can still proceed with hypothesis testing by replacing derivatives with finite differences. Theorem 7.4 in Newey and McFadden (1994) suggests that the numerical derivative for $\frac{1}{T}\sum_{t=1}^{T} q(z_t, x)$ will asymptotically converge in

[34] Lipschitz continuity is also exploited in Theorem 7.3 in Newey and McFadden (1994) as a primitive condition to show stochastic equicontinuity.

probability to the derivative of $\mathbb{E}[q(z_t, x)]$. For instance, let e_i be the i^{th} unit vector whose elements are all zeros except for the i^{th} element which is 1. Also, let h_T a small positive constant that depends on sample size T. If $h_T \to 0$, as $h_T\sqrt{T} \to 0$ and the conditions in Theorem 7.2 of Newey and McFadden (1994) are satisfied, then

$$\frac{1}{T}\sum_{t=1}^{T}\frac{q(z_t, x) - q(z_t, x + e_i h_T)}{h_T} \xrightarrow{p} G_i, \tag{33}$$

in which G_i is the i^{th} element of the gradient of $\mathbb{E}[q(z_t, x)]$. According to this theorem, I can obtain consistent asymptotic variance estimators using finite differences. However, a practical problem with numerical derivatives is the choice of the perturbation parameter h_T used in the denominator. Unfortunately, econometric theory does not provide a clear answer to this problem.

Appendix C Bootstrapped test statistics for the GDA discount factor

In this section, I discuss the bootstrap methodology used to obtain confidence intervals and test statistics for the disappointment model. Let $z_t = \left[\Delta c_{t+1}, \Delta c_t, \{R_{i,t+1}\}_{i=1}^{n-1},\ R_{f,t+1}\right]$. My sample consists of $\{z_1, ..., z_T\}$ observations. As in Hall and Horowitz (1996), I assume that z_t is stationary, ergodic, and that $\mathbb{E}[z_t z_{t+k}] = 0$ for some $k < +\infty$. Stationarity, ergodicity, and weak time-series dependence are reasonable properties for consumption growth, the risk-free rate, and stock returns at the annual frequency.

Following Kunsch (1989), I create m blocs of observations from the original data in order to preserve the autocorelation and covariance structures in the data. Each block will have a length equal to l such that $k \cdot l = T$ where k is an integer. I set the length of each bootstrap block equal to 40 observations because long blocks of observations are required to accurately estimate autocorrelations and covariances. Horowitz (2001) discusses alternative methods of resampling dependent data as well as optimal selection for m and l.

Let Z_i, $i \in \{1, .., m\}$ be a block of observations from the original data, and in particular let

$$Z_1 = \{z_1, .., z_l\}$$

....

$$Z_m = \{z_{T-l+1}, .., z_T\}.$$

with $l = 40$. Let $\{Z_1^*, ..., Z_B^*\}$ be a collection of B random samples generated by randomly drawing blocks of observations with replacement from the set of the available blocks $Z_1, ..., Z_m$. According to Efron and Tibshirani (1993, p. 162), 1000 replications are usually enough to compute standard errors and percentiles. Each bootstrap sample, Z_b^*, is the union of two blocks, $Z_b^* = Z_i \cup Z_j$, and has length $T = 80$.

Let $z_t^{(b)}$ be the t^{th} observation of the bootstrap sample b. For each bootstrap sample, we minimize the GMM objective function, and obtain parameter estimates according to

$$\hat{x}_T^{(b)} = argmin_x \big[\frac{1}{T}\sum_{t=1}^{T} q(z_t^{(b)}, x) - \frac{1}{T}\sum_{t=1}^{T} q(z_t, \hat{x}_T)\big]' \hat{W} \big[\frac{1}{T}\sum_{t=1}^{T} q(z_t^{(b)}, x) - \frac{1}{T}\sum_{t=1}^{T} q(z_t, \hat{x}_T)\big]. \quad (34)$$

where $\hat{x}_T$ are the GMM estimates in the original sample. Following Hall and Horowitz (1996), the bootstrap version of the bootstrap GMM objective function in (34) is recentered relative to the population version to make sure that the bootstrap implements moment conditions that hold in the population. After the minimization, the bootstrap process will result in a bootstrap collection of parameter estimates $(\hat{x}_T^{(1)}, ..., \hat{x}_T^{(1000)})$ and a bootstrap collection of GMM errors $\Big(\frac{1}{T}\sum_{t=1}^{T} q(z_t^{(1)}, \hat{x}_T^{(1)}), ..., \frac{1}{T}\sum_{t=1}^{T} q(z_t^{(1000)}, \hat{x}_T^{(1000)})\Big)$.

In order to show asymptotic normality of non-continuous GMM estimators in Appendix B, we assumed that

$$\sqrt{T}\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x_0)\big] \xrightarrow{d} N(0, \Sigma), \quad (35)$$

and therefore

$$T\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t,x_0)\big]'\Sigma^{-1}\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t,x_0)\big] \xrightarrow{d} X^2_{(K)},$$

where $X^2_{(K)}$ is the chi-square distribution with K degrees of freedom, and K is the length of $q(z_t, x_0)$. According to Lemma 4.1 in Hansen (1982) and p. 210 in Cochrane (2001), the covariance matrix for the first-stage GMM errors is given by

$$\Sigma^{1st\ GMM} = \Big(I - G\big(G'WG\big)^{-1}G'W\Big)\Sigma\Big(I - G\big(G'WG\big)^{-1}G'W\Big)',$$

where G is the gradient of $\mathbb{E}\big[q(z_t, x_0)\big]$, and Σ is the asymptotic variance-covariance matrix of $\sqrt{T}\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, x_0)\big]$ if we did not estimate any parameters (Cochrane, p. 204).

If $\hat{\Sigma}$ and $\hat{G}$ are consistent estimates of S and G respectively, then

$$\hat{\Sigma}^{1st\ GMM} = \Big(I - \hat{G}\big(\hat{G}'\hat{W}\hat{G}\big)^{-1}\hat{G}'\hat{W}\Big)\hat{\Sigma}\Big(I - \hat{G}\big(\hat{G}'\hat{W}\hat{G}\big)^{-1}\hat{G}'\hat{W}\Big),$$

which implies that

$$\sqrt{T}\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t,\hat{x}_T)^{1st\ GMM}\big] \xrightarrow{d} N(0, \hat{\Sigma}^{1st\ GMM}),$$

and

$$T\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t,\hat{x}_T)^{1st\ GMM}\big]'\big(\hat{\Sigma}^{1st\ GMM}\big)^{-1}\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t,\hat{x}_T)^{1st\ GMM}\big] \xrightarrow{d} X^2_{(K-L)},$$

where L is the number of parameters to be estimated.

When the number of observations is limited, asymptotic arguments fail. For instance, estimates of the covariance matrix $\Sigma^{1st\ GMM}$ could be biased, and $\hat{\Sigma}^{1st\ GMM}$ would be singular. Furthermore, since the disappointment model $q(z_t, x_0)$ is not differentiable, $\hat{\Sigma}^{1st\ GMM}$ will depend on the choice of the perturbation parameter h_T for the finite differences approximation of $\hat{G}$ (equation (33) in

Appendix B). To a large extend all these issues can be addressed, if we use the bootstrap sample.

First, we can obtain an estimate of Σ in (35) from the bootstrap sample as follows

$$\hat{\Sigma}_{boot} = T\widehat{Var}\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t^{(b)}, \hat{x}_T^{(b)})^{1st\ GMM}\big].$$

We can then estimate $\hat{\Sigma}_{boot}^{1st\ GMM}$ as

$$\hat{\Sigma}_{boot}^{1st\ GMM} = \Big(I - \hat{G}_{boot}\big(\hat{G}_{boot}'\hat{W}\hat{G}_{boot}\big)^{-1}\hat{G}_{boot}'\hat{W}\Big)\hat{\Sigma}_{boot}\Big(I - \hat{G}_{boot}\big(\hat{G}_{boot}'\hat{W}\hat{G}_{boot}\big)^{-1}\hat{G}_{boot}'\hat{W}\Big),$$

in which $\hat{G}_{boot}$ is the bootstrap estimate of the gradient of $\mathbb{E}\big[q(z_t, x_0)\big]$. Finally, we can proceed with hypothesis testing using the following statistics:

1. testing the entire set of moment conditions:

$$T\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, \hat{x}_T)^{1st\ GMM}\big]'\big(\hat{\Sigma}_{boot}^{1st\ GMM}\big)^{*-1*}\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, \hat{x}_T)^{1st\ GMM}\big] \xrightarrow{d} X^2_{(K-L)},$$
$$\text{under } H_0: \ \mathbb{E}\big[q(z_t, x_0)\big] = 0,$$

2. testing subsets of moment conditions:

$$\sqrt{T}\mathbf{A}\big[\frac{1}{T}\sum_{t=1}^{T} q(z_t, \hat{x}_T)^{1st\ GMM}\big] \xrightarrow{d} N(0, \mathbf{A}\hat{\Sigma}_{boot}^{1st\ GMM}\mathbf{A}'), \text{ under } H_0: \ \mathbf{A}\mathbb{E}\big[q(z_t, x_0)\big] = 0,$$
with $\mathbf{A}$ an $M \times K$ matrix,

3. testing individual moment conditions:

$$(\sqrt{T}\frac{1}{T}\sum_{t=1}^{T} q_i(z_t, \hat{x}_T)^{1st\ GMM})\sqrt{\big(\hat{\Sigma}_{boot}^{1st\ GMM}\big)_{ii}^{*-1*}} \xrightarrow{d} N(0,1), \text{ under } H_0: \ \mathbb{E}[q_i(z_t, x_0)] = 0.$$

The exponent $*-1*$ denotes the Moore-Penrose pseudo-inverse that adjusts the bootstrap covariance estimator for recentering in (34), and for the fact that the blocks may not exactly replicate the dependence structure of the data (Hall and Horowitz (1996), Chou and Zhou (2006)).

Finally, if $\hat{x}_T^{(b)}$ is a bootstrap estimate, then, as long as $\hat{x}_T - x_0 \sim \hat{x}_T^{(b)} - \hat{x}_T$, we can test the significance of the GMM estimates of x_0 using confidence intervals that are based on the distribution of the bootstrapped estimates $\hat{x}_T^{(b)}$

$$\hat{\mathbb{P}}\big(x_0 \in [\hat{x}_{T,a^*\%}^{(b)},\ \hat{x}_{T,1-a^*\%}^{(b)}]\big)_{boot} = 1 - 2a\%.$$

The confidence intervals $[\hat{x}_{T,a^*\%}^{(b)},\ \hat{x}_{T,1-a^*\%}^{(b)}]$ are corrected for bias using the bias-corrected method in Efron and Tibshirani (1986).

Appendix D The relation between GDA preferences and safety-first portfolios

In this section, I show that disappointment aversion can be mapped into safety-first or conditional value-at-risk portfolio problems.

Let $\mathbf{R} \in [-1, +\infty)^n$ be a random vector of n asset returns and $F_{\mathbf{R}}$ be the multivariate c.d.f. associated with these returns. Let $\mathbf{w} \in \mathbb{R}$ be the vector of n portfolio weights. Define the first lower partial moment of the investment portfolio as

$$\mathbb{E}\big[(\mathbf{w}'\mathbf{R} - \mu)\mathbf{1}\{\mathbf{w}'\mathbf{R} \leq \mu\}\big] = \int_{-\infty}^{\mu} (\mathbf{w}'\mathbf{R} - \mu) dF_{\mathbf{R}},$$

with μ equal to

$$\mu = \mathbb{E}\big[\mathbf{w}'\mathbf{R}\frac{1 + \lambda\mathbf{1}\{\mathbf{w}'\mathbf{R} \leq \mu\}}{1 + \lambda\mathbb{E}\big[\mathbf{w}'\mathbf{R} \leq \mu\big]}\big], \tag{36}$$

for some positive parameter λ.

Now, consider the following portfolio choice problem under Bawa's (1978) safety-first criterion.

$$\max_{\mathbf{w}} \mathbb{E}[\mathbf{w}'\mathbf{R}] \quad s.t.$$
$$\mathbb{E}\big[(\mathbf{w}'\mathbf{R} - \mu)\mathbf{1}\{\mathbf{w}'\mathbf{R} \leq \mu\}\big] = 0 \tag{37}$$
$$\sum_{i}^{n} w_i = 1.$$

The above problem is equivalent to a portfolio choice problem in which the objective function is based on conditional value-at-risk criteria (Rockafellar and Uryasev (2000)).

Using the Lagrange multiplier $\theta > 0$ for the safety-first constraint in equation (37), the portfolio problem can be written as

$$\max_{\mathbf{w}} \Big\{\mathbb{E}[\mathbf{w}'\mathbf{R}] + \theta\mathbb{E}\big[(\mathbf{w}'\mathbf{R} - \mu)\mathbf{1}\{\mathbf{w}'\mathbf{R} \leq \mu\}\big]\Big\} \quad s.t.$$
$$\sum_{i}^{n} w_i = 1.$$

If we assume that the positive parameter λ in equation (36) is equal to the Lagrange multiplier θ, the above problem is equivalent to the portfolio problem for an investor who is disappointment averse, and whose preferences can be described by a utility function of the form

$$U(x) = \begin{cases} x, \ x > \mu \\ x + \theta(x - \mu), \ x \leq \mu \end{cases}$$

with μ given in equation (36) for $\lambda = \theta$.